The University of British Columbia

Final Examination - Math 257/316
Tuesday, April 14, 2009, 8:30-11:00am

Closed book examination — Time: 2.5 hours

Last Name: ________________ **Signature:** ______________________

First Name: ________________ **Student Number:** ________________

Course number: ________________

Do not open this test until instructed to do so! This exam should have **7 pages**, including this cover sheet. No textbooks, calculators, or other aids are allowed. Turn off any cell phones, pagers, etc. that could make noise during the exam. **Circle your solutions! Reduce your answer as much as possible. Explain your work. Relax.** Use the back of the page if necessary.

Read these UBC rules governing examinations:

(i) Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.

(ii) Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

(iii) No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

(iv) Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

- Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
- Speaking or communicating with other candidates.
- Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

(v) Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Problem	**Out of**	**Score**
1	15	
2	20	
3	20	
4	15	
5	20	
6	10	
Total	100	

Name:____________________

Problem 1 (total 15 points) Let

$$f(x) = \begin{cases} 0 & \text{if } -1 \leq x \leq 0 \\ 1 - x & \text{if } 0 < x \leq 1 \end{cases}$$

with $f(x+2) = f(x)$

(a) [10] Find the Fourier series corresponding to the given function $f(x)$.

(b) [5] Sketch the graph of the function to which the series converges over two periods.

Name:____________________

Problem 2 (20 points)

Consider the differential equation

$$x^2y'' + xy' + \left(x^2 - \frac{9}{4}\right) y = 0, \qquad x > 0.$$

(a) [5] Classify the point $x_0 = 0$ as ordinary point, regular singular point, or irregular singular point.

(b) [10] Find two values of r such that there are solutions of the form $y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}$ and find the recurrence relation for a_n in dependence on r.

(c) [5] For the larger of the two values of r and for $a_0 = 1$ find the first 3 nonzero terms of the solution.

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Problem 3 (20 points)

Find the solution $u(x,t)$ of the following heat equation

$$\begin{aligned} &u_t = 9u_{xx}, \quad 0 < x < 1,\ t > 0 \\ &u(0,t) = 2, \quad u(1,t) = 0 \\ &u(x,0) = -2x \end{aligned}$$

(a) [5] Find the steady-state solution $v(x)$.

(b) [15] Find the solution $u(x,t)$.

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Problem 4 (15 points)

Find the solution $u(r,\theta)$ of the following Laplace equation in a pie-shaped domain:

$$
\begin{aligned}
&u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0, \qquad 0 < r < 1,\ 0 < \theta < \frac{\pi}{3},\\
&u_\theta(r,0) = u_\theta(r,\frac{\pi}{3}) = 0,\\
&u(r,\theta) \text{ bounded for } r \to 0,\\
&u_r(1,\theta) = f(\theta) = \begin{cases} 1 & \text{if } 0 < \theta < \frac{\pi}{6},\\ -1 & \text{if } \frac{\pi}{6} < \theta < \frac{\pi}{3}. \end{cases}
\end{aligned}
$$

(a) [5] Find the Fourier cosine series of $f(\theta)$ for $0 < \theta < \frac{\pi}{3}$.

(b) [10] Find $u(r,\theta)$.

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Problem 5 (20 points)

The steady-state temperature distribution $u(x, y)$ of a square plate with one side held at $100°$ and the other three sides at $0°$ satisfies the following Laplace equation:

$$\begin{aligned}
&u_{xx} + u_{yy} = 0, \qquad 0 < x < \pi,\ 0 < y < \pi, \\
&u(0, y) = 0, \quad u(\pi, y) = 100, \\
&u(x, 0) = 0, \quad u(x, \pi) = 0.
\end{aligned}$$

Find the solution $u(x, y)$.

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Problem 6 (10 points)

The vibrations of a damped string of length 1 are described by the damped wave equation:

$$\begin{aligned}
&u_{tt} + 2\pi u_t = u_{xx}, \qquad 0 < x < 1,\ t > 0\\
&u(0,t) = 0, \quad u(1,t) = 0,\\
&u(x,0) = \sin(3\pi x),\\
&u_t(x,0) = 0,
\end{aligned}$$

The solution is of the form $u(x,t) = T(t)\sin(3\pi x)$. (You don't have to show this.)

(a) [5] Find the real-valued general form of $T(t)$.

(b) [5] Find the real-valued solution $u(x,t)$.