Probabilistic Models of Critical Phenomena

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Clarification May 6, 2009

The following formulas are stated in Sections 3.2–3.3:

\[ N \sim \begin{cases} 
2\epsilon^{-2} \log n & \text{if } \epsilon < 0 \\
A n^{2/3} & \text{if } \epsilon = 0 \\
2\epsilon n & \text{if } \epsilon > 0,
\end{cases} \]

the second largest cluster has size \(2\epsilon^{-2} \log n\), and

\[ \chi(p) \sim \begin{cases} 
1/|\epsilon| & \text{if } \epsilon < 0 \\
c n^{1/3} & \text{if } \epsilon = 0 \\
4\epsilon^2 n & \text{if } \epsilon > 0.
\end{cases} \]

The constants \(2\epsilon^{-2}, 2\epsilon, 2\epsilon^{-2}, 4\epsilon^2\) in these formulas are not correct when \(\epsilon\) is fixed. Different constants occur for fixed \(\epsilon\). The stated formulas are correct when \(\epsilon \to 0\) as \(n \to \infty\), but not too quickly, with \(|\epsilon| n^{1/3} \to \infty\) (i.e., outside the so-called critical window). The formula \(1/|\epsilon|\) for \(\chi(p)\) when \(\epsilon < 0\) is correct both for fixed \(\epsilon\) and in the above mentioned limit of small \(\epsilon\).

I am grateful to Doron Zeilberger and the students of his Experimental Mathematics class at Rutgers University for bringing this to my attention.