**Last time: Left Riemann sums**

\[ x_0 = a \quad x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_{n-1} \quad x_n = b \]

width = \( \Delta x = \frac{b-a}{n} \)

Left Riemann sum = \( \Delta x \cdot \left[ f(x_0) + f(x_1) + \ldots + f(x_{n-1}) \right] \)

\[ = \sum_{i=0}^{n-1} \Delta x \cdot f(x_i) \]

\[ = \sum_{i=0}^{n-1} f(x_i) \Delta x \]

Right Riemann sum = \( \Delta x \cdot \left[ f(x_1) + f(x_2) + \ldots + f(x_n) \right] \)

\[ = \sum_{i=1}^{n} f(x_i) \Delta x \]

As \( n \to \infty \), the Left and the Right Riemann Sums converge and we get the true area under the curve.
The point is that this is how we define the area under the graph of \( y = f(x) \):

\[
\int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} \sum_{i=0}^{n-1} f(x_i) \Delta x = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x
\]

Area under the curve \( f(x) \) between \( a \leq x \leq b \)

Left and Right Riemann sums converge to the same limit.

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**Question:** What is \( \int_{1}^{4} 5 \, dx \)?

**Answer:** It is the area underneath the graph of \( f(x) = 5 \) for \( 1 \leq x \leq 4 \).

\[
\int_{1}^{4} 5 \, dx = (4-1) \cdot 5 = 3 \cdot 5 = 15
\]
Question: What about \( \int_{-3}^{4} 2 \, dx \)?

\[
\text{Area} = 2 \cdot (4 - (-3)) = 2 \cdot 7 = 14
\]

So, \( \int_{-3}^{4} 2 \, dx = 14 \).

We see that for a constant function \( f(x) = k \), finding area underneath reduces to the problem of finding an area of a rectangle.

\[
\int_{a}^{b} k \, dx = k \cdot (b - a)
\]

Suppose we knew that \( \int_{1}^{3} x^2 \, dx = \frac{26}{3} \).

(We will later learn how to calculate this).

What can we say about \( \int_{1}^{3} 2x^2 \, dx \)?

Well, the graph of \( g(x) = 2x^2 \) is vertically scaled from the graph of \( f(x) = x^2 \) by a factor of 2.

\[
\int_{1}^{3} x^2 \, dx = \frac{26}{3}
\]

\[
\int_{1}^{3} 2x^2 \, dx = 2 \left( \frac{26}{3} \right) = \frac{52}{3}
\]
General rule: \[ \int_a^b c \cdot f(x) \, dx = c \int_a^b f(x) \, dx \]
whenever "c" is a constant.

**Question:** Find \[ \int_1^3 (x^2 + 3) \, dx. \]

We will take advantage of the fact that \[ \int_1^3 x^2 \, dx = \frac{26}{3} \]
[given to us in previous page] We also know \[ \int_1^3 3 \, dx = 3 \cdot (3 - 1) = 3 \cdot 2 = 6. \]

In other words, \[ \int_1^3 (x^2 + 3) \, dx = \int_1^3 x^2 \, dx + \int_1^3 3 \, dx = \frac{26}{3} + 6 = \frac{44}{3}. \]

**General Rule:** \[ \int_a^b f(x) \, dx + \int_a^b g(x) \, dx = \int_a^b (f(x) + g(x)) \, dx \]

Suppose we know that \[ \int_2^4 x^3 \, dx = 60. \] Find \[ \int_2^4 (3x^3 + 7) \, dx. \]

**Answer:** \[ \int_2^4 3x^3 \, dx + \int_2^4 7 \, dx = 3 \int_2^4 x^3 \, dx + \int_2^4 7 \, dx \]
\[ = 3 \cdot 60 + 7(4 - 2) = 180 + 14 = 194. \]