Lines: We want to understand equations of lines.

Slope of a line: The slope is a quantity that measures how steep the line is, as you go from left to right. Consider a line segment that joints the points \( P=(x_1, y_1) \) and \( Q=(x_2, y_2) \).

\[
\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}.
\]

Sometimes, we also write \( \frac{\Delta y}{\Delta x} \).

In general, \( \Delta \) in math usually denotes "change in a quantity".

Here, \( \Delta y = y_2 - y_1 \) and \( \Delta x = x_2 - x_1 \). Read \( \Delta \) as "delta" (Greek letter).

Examples: Find the slopes of the following lines:

A) \[
\text{Slope} = \frac{4 - 2}{3 - 1} = \frac{2}{2} = 1
\]

B) \[
\text{Slope} = \frac{2 - 2}{2 - 1} = \frac{0}{1} = 0.
\]

C) \[
\text{Slope} = \frac{2 - 5}{1 - (-2)} = \frac{-3}{3} = -1
\]

D) \[
\text{Slope} = \frac{3 - 1}{1 - 1} = \frac{2}{0}, \quad \text{is undefined!}
\]

(In some sense, the slope is \( \infty \) here.)
Equation of the line

To find the equation of a line, we need the following two pieces of information:

(1) slope of the line \( m \).
(2) a point on the line, say, \( P = (x_1, y_1) \).

Then, the equation of the line is:

\[
y - y_1 = m(x - x_1) \rightarrow \text{slope-point formula.}
\]

Example: What is the equation of a line with slope \( m = -2 \), and the point \((1, 2)\) on the line?

\[
m = -2, \quad P = (x_1, y_1) = (1, 2), \quad \text{so} \quad x_1 = 1, \quad y_1 = 2.
\]

\[
y - 2 = -2(x - 1)
\]

Slope = -2 means that for every 1 unit to right results in 2 units down.

We can rearrange the equation \( y - 2 = -2(x - 1) \):

\[
y = 2 - 2(x - 1) \quad \rightarrow \quad y = -2x + 2 \quad \rightarrow \quad y = -2x + 4
\]

\[
y = mx + b \rightarrow \text{slope-intercept formula.}
\]

The value "b" is the y-intercept of line. This means \( b \) is where the line meets the y-axis.

You can check this by plugging in \( x = 0 \):

\[y = m \cdot 0 + b = b\]
Example: What is the equation of the line passing through \((-1, 2)\) and \((2, 5)\)?

Solution: We need to find:

1) slope \(m\)

2) a single point \(P=(x_1, y_1)\) on the line.

1) \[
slope = \frac{\text{rise}}{\text{run}} = \frac{y_2-y_1}{x_2-x_1} = \frac{5-2}{2-(-1)} = \frac{3}{3} = 1
\]

\(P=(x_1, y_1)=(-1, 2)\)

\(Q=(x_2, y_2)=(2, 5)\)

2) We take \(P=(x_1, y_1)=(-1, 2)\).

So, the slope-point form of the line is:

\[
y-2 = 1 \cdot (x-(-1))
\]

\[
y-2 = x+1 \implies y=x+3.
\]

Example: Find the slope and the \(y\)-intercept of a line given by the equation \(2x+3y=12\).

Solution: We need to transform \(2x+3y=12\) into the format \(y=mx+b\)

\[
slope = \frac{y-intercept}{y-intercept}.
\]

\[
2x+3y=12 \implies 3y=-2x+12 \implies y=\left(-\frac{2}{3}\right)x+4.
\]

So,

\[
slope = -\frac{2}{3}, \quad y\text{-intercept}=4.
\]