1. **3 marks** Let $V$ be an $\mathbb{R}$-vector space and let $V^*$ be its dual space. Recall that we have a bilinear map $\Psi : V^* \times W \rightarrow \text{Hom}(V, W)$ sending a pair $f, w$ to the map $V \rightarrow W$ sending $v \in V$ to $f(v) \cdot w$. Is the image of this map, i.e. the set of all linear maps $V \rightarrow W$ that can be written in the form $f(-) \cdot w$ for some $f \in V^*$ and some $w \in W$, a subspace of $\text{Hom}(V, W)$?

2. **3 marks** Compute the determinant of the following matrix:

$$
\begin{pmatrix}
1 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 \\
\end{pmatrix}
$$

3. **4 marks** Let $F = \{a+b\sqrt{2}+c\sqrt{3}+d\sqrt{6} \mid a, b, c, d \in \mathbb{Q}\}$. $F$ is a subfield of the real numbers. If we consider $F$ as a $\mathbb{Q}$-vectorspace, $F$ is 4-dimensional and a basis is given by $\{1, \sqrt{2}, \sqrt{3}, \sqrt{6}\}$. Left multiplication with $\sqrt{2} + \sqrt{3}$ is a $\mathbb{Q}$-linear map $f : F \rightarrow F$.

1. Find the matrix of $f$ with respect to the upper basis.
2. Compute the determinant of that matrix.