1. 4 marks Are the following vectors linearly independent?
\[
\begin{pmatrix}
2 \\
1 \\
-3 \\
\end{pmatrix},
\begin{pmatrix}
1 \\
0 \\
-1 \\
\end{pmatrix},
\begin{pmatrix}
2 \\
-1 \\
-1 \\
\end{pmatrix}
\]

**Solution:** No. We have:
\[
\begin{pmatrix}
2 \\
1 \\
-3 \\
\end{pmatrix} + (-4) \cdot \begin{pmatrix}
1 \\
0 \\
-1 \\
\end{pmatrix} + \begin{pmatrix}
2 \\
-1 \\
-1 \\
\end{pmatrix} = 0
\]

How can you see that? If you look at the 2nd coordinate, you see that the coefficient of the first and the last vector have to be the same.

2. 4 marks Let \( V \) be the set of pairs of real numbers \((a, b)\) with \( b > 0 \). Define an addition on \( V \) via:
\[
(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2b_2)
\]
and a scalar multiplication by \( c \cdot (a_1, a_2) = (ca_1, a_2^c) \). Is \( V \) a vectorspace with these operations?

**Solution:**
- \((a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2b_2) = (b_1 + a_1, b_2a_2) = (b_1, b_2) + (a_1, a_2)\).
- \([(a_1, a_2) + (b_1, b_2)] + (c_1, c_2) = (a_1 + b_1, a_2b_2) + (c_1, c_2) = (a_1 + b_1 + c_1, a_2b_2c_2) = (a_1 + b_1 + c_1, a_2b_2c_2) = (a_1, a_2) + (b_1 + c_1, b_2c_2) = (a_1, a_2) + ((b_1, b_2) + (c_1, c_2))\).
- The zero is \((0, 1)\). We have: \((a, b) + (0, 1) = (a + 0, b \cdot 1) = (a, b)\).
- The inverse of \((a, b)\) is \((-a, 1/b)\):
\[
(a, b) + (-a, 1/b) = (a - a, b \hat{1}/b) = (0, 1)
\]
- \(1 \cdot (a, b) = (1a, b^1) = (a, b)\).
- \((rs)(a, b) = (rsa, b^{rs}) = r(sa, b^s) = r(s(a, b))\).
- \(r((a_1, a_2) + (b_1, b_2)) = (ra_1 + ra_2, (a_1a_2)r) = (ra_1 + ra_2, a_1^ra_2^r) = r(a_1, a_2) + r(b_1, b_2)\).
- \((r+s)(a, b) = ((r+s)a, b^{r+s}) = (ra + sa, b^r b^s) = (ra, b^r) + (sa, b^s) = r(a, b) + s(a, b)\).

So it is a vectorspace.
3. Let $V$ be set of pairs of real numbers. Define an addition via:

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 b_2)$$

and a scalar multiplication via

$$c \cdot (a_1, a_2) = (ca_1, a_2).$$

Is $V$ a vectorspace with these operations?

**Solution:** The neutral element with respect to addition is $(0, 1)$. So there has to be an inverse of $(0, 0)$. But we have: $(0, 0) + (a, b) = (a, 0) \neq (0, 1)$. 