1. Show that \( F(z) = \log(-z) + i\pi \) is a branch of \( \log(z) \) with branch cut on the positive real axis. Is it true that \( F(z) = \log_+(z) \)? (Hint: don’t forget to check values right on the cut)

Recall that for \( z \in \mathbb{C}\setminus\{0\} \) and \( \alpha \in \mathbb{C} \), the complex power \( z^\alpha \) is defined to be \( z^\alpha = \exp(\alpha \log(z)) \) (as a multivalued function).

2. Show that \( (zw)^\alpha = z^\alpha w^\alpha \) as sets. (The set on the right is \( \{a \cdot b : a \in z^\alpha, b \in w^\alpha\} \})

3. Show that \( z^\alpha \)
   (a) is single valued if \( \alpha \in \mathbb{Z} \),
   (b) has \( q \) values if \( \alpha = p/q \), where \( p, q \in \mathbb{Z} \) with no common factors and \( q > 0 \).
   (c) has infinitely many values if \( \alpha \) is irrational.

4. Identify the branch points of \( f(z) = \log(z(z + 1)/(z - 1)) \). (Don’t forget to check \( z = \infty \).)
   If we define a branch for \( f(z) \) by choosing the principal branch of \( \log(z) \), where are the branch cuts? (Note: this example illustrates that there may be a choice of branch cuts not obeying our “contractible loops” condition that still result in a single valued function.)

5. Find the branch points of \( f(z) = (z^3 + z^2 - 6z)^{1/2} \). Define a branch \( F(z) \) using the “range of angles” method that is continuous at \( z = -1 \) with \( F(-1) = -\sqrt{6} \).

6. Construct a branch \( F(z) \) of \( (z^2 + 1)^{1/2} \) that is
   (i) analytic inside the unit circle,
   (ii) analytic away from the imaginary axis,
   (iii) equals \( \sqrt{x^2 + 1} \) for \( x \in \mathbb{R} \).
   (iv) is continuous on the imaginary axis from the right.
Give an algorithm (i.e., a sequence of steps) that takes as input two real numbers \( x \) and \( y \) and computes \( F(x + iy) \)