Math 257/316, Midterm 1, Section 101
9 am on 18th of October 2017

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed. A formula sheet is provided.

Maximum score 50.

1. Consider the second order differential equation:

\[ Ly = 2x^2 y'' + 3xy' + (2x - 1)y = 0 \]  \hspace{1cm} (1)

(a) Classify the points \(0 \leq x < \infty\) as ordinary points, regular singular points, or irregular singular points. For any regular singular points determine the roots of the corresponding indicial equation. \[7\text{ marks}\]

(b) If you were given \(y(-1) = 1\) and \(y'(-1) = 0\), what form of series expansion would you assume (Do not determine the expansion coefficients of this series)? What would be the minimal radius of convergence of this series?

\[3\text{ marks}\]

(c) Use the appropriate series expansion about the point \(x = 0\) to determine two independent solutions to (1). You only need to determine the first three non-zero terms in each case. What is the minimal radius of convergence of these series?

\[20\text{ marks}\]

2. Apply the method of separation of variables to determine the temperature \(u(x, t)\) in a rod of length \(\pi/2\) that involves a chemical reaction that generates heat at a rate proportional to the temperature, maintained at a zero temperature at the left endpoint, and insulated at the right endpoint. The initial-boundary value problem satisfied by \(u(x, t)\) is given by:

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \gamma^2 u, \quad 0 < x < \pi/2, \ t > 0
\]

BC : \(u(0, t) = 0\) and \(0 = \frac{\partial u(\pi/2, t)}{\partial x}\)

IC : \(u(x, 0) = \sin 3x\)

Please show all the cases when solving the appropriate eigenvalue problem.

Hint: When separating the variables, group the \(\gamma^2\) term with the time ordinary differential equation.

\[20\text{ marks}\]