Math 257-316: Assignment 2’2018

Note: This assignment is due at the beginning of class of Wednesday, September 19th.

[Power series solutions of ODE]

1. [Will be marked] Consider the ODE:

\[ y'' - 2xy' + y = 0 \]

Compute the first 3 nonzero terms of power series expansion about \( x = 0 \) for two linearly independent solutions. Use the ratio test to determine the radius of convergence of the series. Could your result have been predicted by inspection?

2. [Will be marked] Consider the Differential equation \( y'' + x^2y' - y = 0 \). Find the first 3 terms of the power series expansion of the general solution about \( x = 0 \). Now determine the solution that satisfies the initial conditions \( y(0) = 0 \) and \( y'(0) = 1 \).

3. Compute the first 3 nonzero terms of the power series expansion about \( x = 0 \) of two linearly independent solutions of the ODE:

\[ y'' - (\sin x) y = 0 \]

4. Find the power series expansion about \( x = 0 \) of two linearly independent solutions of the ODE

\[ y'' + x^2y = 0 \]

What can you say about the radius of convergence of this power series?

[Regular Singular points of ODE and Frobenius Series]

5. Find all the singular points of the following ODEs and determine whether each one is regular or irregular. If the singular point is regular, determine the indicial equation and determine a lower bound for the radius of convergence of the Frobenius series. You must not try to compute the series solution itself.

a. \((x^2 - 1)y'' + x^2y' + \cot(x)y = 0\).

b. \(\sinh(x)y'' + xy' + y = 0\).

c. \((x^2 + 4)y'' + xy' + y = 0\).

d. \(y'' + x^{1/2}y' - y = 0\).