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APPENDIX: NOTATION

The following symbols are used in this paper.
A = air filled voids,
C_0 = bar wave speed,
E_{bar} = bar modulus,
L_0 = sample thickness,
P = porosity,
S = saturation,
V_{ex} = excitation voltage,
V_p = projectile velocity,
$V(t)$ = signal voltage,
ε_i = incident strain,
ε_r = reflected strain,
ε_t = transmitted strain,
ε_s = sample strain,
$\varepsilon(t)$ = strain calculated from the voltage signals,
$\dot{\varepsilon}_s$ = sample strain rate,
$\rho(av)$ = average tuff density,
$\rho(dry)$ = density of the dry tuff samples,
$\rho(sat)$ = density of the saturated tuff samples,
$\rho(solid)$ = solid tuff density,
σ_s = sample stress.

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On the numerical stability of time domain boundary element methods

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ABSTRACT: Time domain elastodynamic boundary element methods are prone to numerical instabilities. Under suitable conditions, these instabilities can swamp the transient response of a system. We show evidence of these instabilities in both the direct and indirect boundary element methods. We summarize the literature on the evidence and causes of these instabilities, and refer to improved algorithms and alternative formulations which are less prone to numerical instabilities. Finally, we make suggestions as to where research should concentrate so that these methods can reach their full potential.

KEYWORDS: elastodynamics, numerical stability, fundamental principles, boundary element methods, direct, indirect.

INTRODUCTION

Over the past thirty years, a wealth of literature has been published on elastodynamic boundary element methods. Elastodynamic formulations have been developed for the Fourier and Laplace transform domains, as well as for the time domain. Direct and indirect boundary integral approaches have been developed in both two and three dimensions. Variational, domain or mass matrix, T-matrix, and Born approximation formulations have also been developed. The reader is referred to Beskos (1997) for an exhaustive summary of the literature.

Because of numerical difficulties, there has been limited success in developing software of commercial quality in this field, when compared with similar developments using finite difference or finite element techniques. In particular, time domain boundary integral methods are prone to numerical instabilities, which tend to swamp the solution if it is advanced far enough in time. In many cases, these instabilities can quickly swamp the transient behavior so as to render the results useless.

This paper summarizes the literature on the evidence and causes of numerical instabilities in the time domain boundary element methods. New time stepping algorithms are referred to which will delay and often eliminate instabilities in both the direct and indirect methods. Recommendations are made as to where research should concentrate so that these methods can reach their full potential.

THE NUMERICAL STABILITY PROBLEM

Figure 1 shows the geometry and loading of a 28 element crack problem, where the steep side is loaded by prescribed shear tractions and the remaining side is traction free. Figure 2 shows the displacement history at the field point marked in Figure 1, obtained using the two-dimensional displacement discontinuity method code of Siebrits (1992). The figure shows clear evidence of a numerical instability by 250 time steps.

Figure 3 shows a circular cavity in an infinite space, modeled using 16 elements, and loaded suddenly by a prescribed normal traction. Figure 4 shows the radial displacement history, obtained using the two-dimensional direct boundary element code of Dominguez (1993), and an instability is evident by 1800 time steps.

Based on these two simple examples, the reader will question the need to discuss the numerical instability issue because the instability becomes evident after a rela-

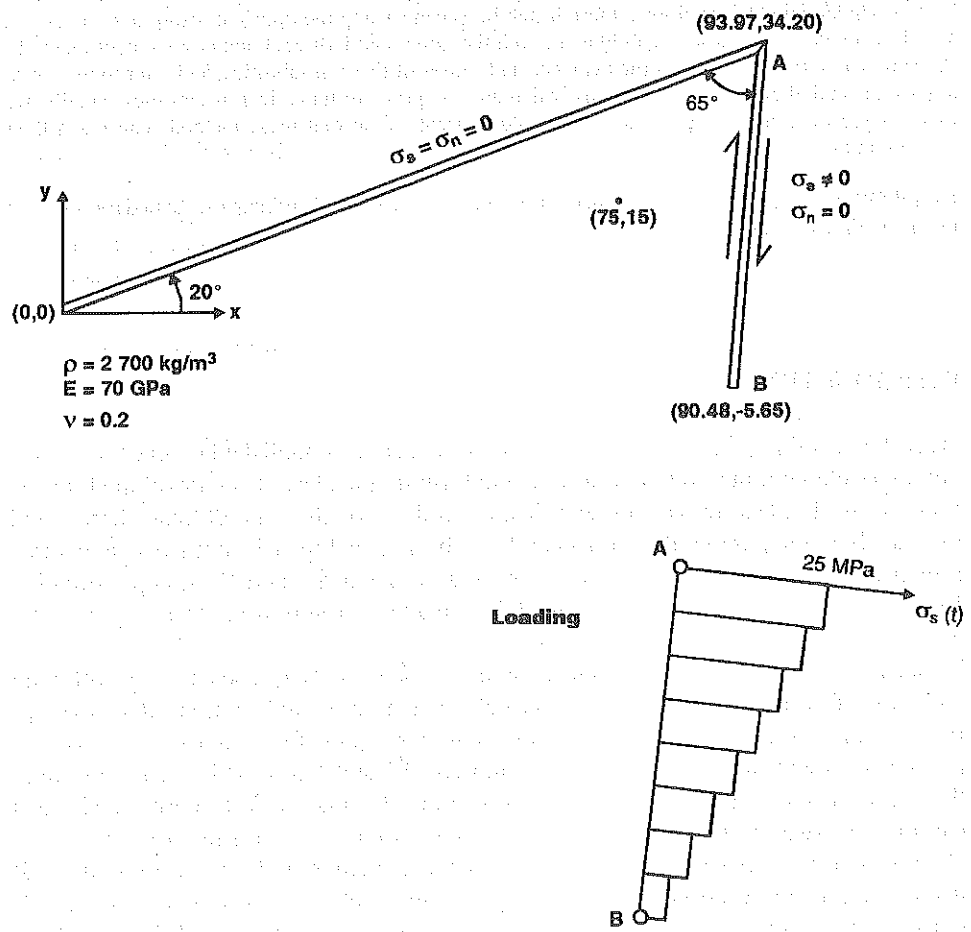

Figure 1. Geometry and loading of a crack problem.

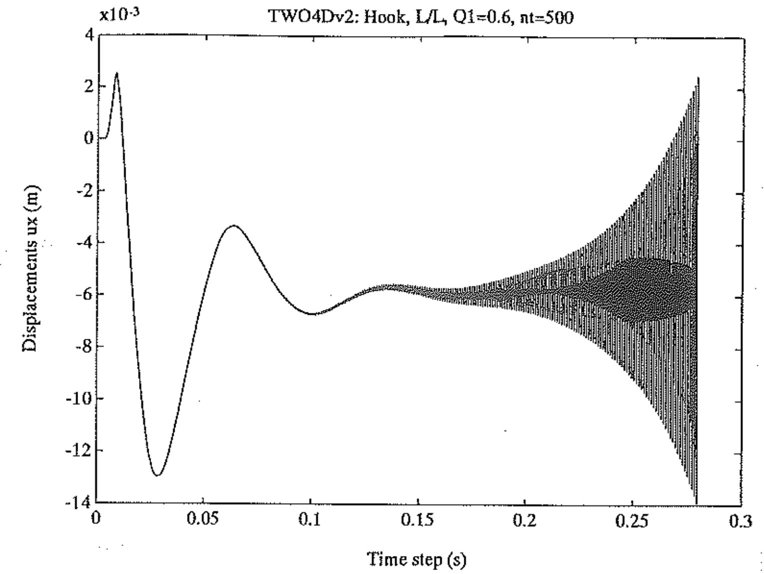

Figure 2. Displacement history showing evidence of a numerical instability in the displacement discontinuity method.

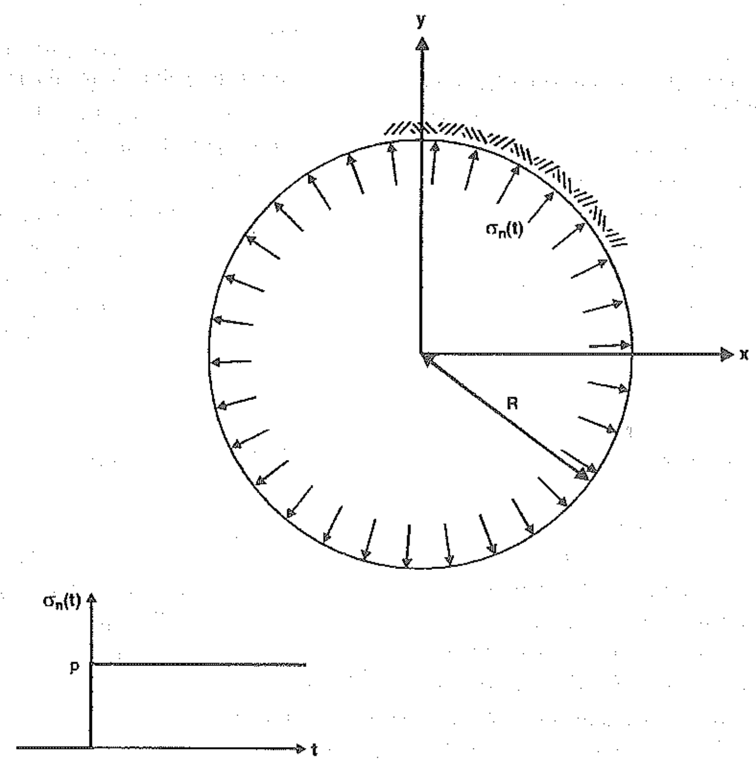

Figure 3. Geometry and loading of a cavity problem.

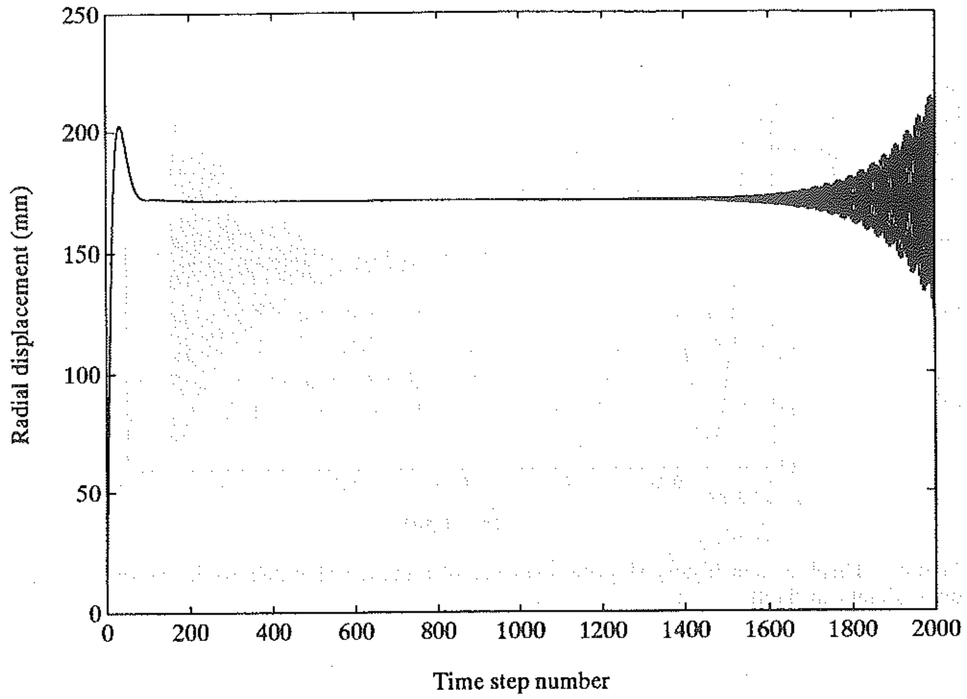

Figure 4. Displacement history showing evidence of a numerical instability in the direct boundary element method.

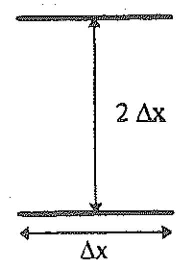

Figure 5. Two element antiplane strain crack problem which exhibits instabilities.

tively large number of time steps, towards the end of, or even after, the transient behavior has occurred. However, these instabilities can become evident during the transient phase if the geometry and/or loading configuration becomes more complicated. For example, if only half of the cavity in Figure 3 is loaded, then instabilities become evident by 300 time steps (Siebrits & Peirce 1997).

These numerical instabilities have also been found to be of an intermittent nature (Peirce & Siebrits 1996, 1997). Figure 5 shows a test case of two crack elements, aligned opposite each other, under antiplane strain conditions. The one element is loaded suddenly using the loading function shown in Figure 3, and the other element

Table 1. Intermittent nature of instabilities in the two element problem.

Q2	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
Stable?	no	no	yes	no	yes	yes	no	yes
Q2	0.90	1.00	1.10	1.20	1.30	1.34	1.40	1.50
Stable?	yes	yes	yes	yes	yes	no	yes	yes

is traction free. This problem was modeled using the displacement discontinuity method under antiplane strain conditions, and the stability results are summarized in Table 1. In Table 1, $Q2 = c_2\Delta t / \Delta x$, where c_2 = shear wave velocity, Δt = time step, and Δx = element length. The table shows that there is not a critical time step associated with stability. In fact, stability is governed by geometric and temporal constraints, and has been shown to depend on how source and receiver points interact with each other dynamically (Peirce & Siebrits 1996, 1997). The authors have found that the two element model problem is a good test of stability in the boundary element algorithms that they have considered. The short run times and ease with which the geometry and run parameters can be changed makes this problem an ideal candidate for other researchers to use as a test for prototype algorithms.

LITERATURE REVIEW

Relatively few papers provide clear evidence of numerical instabilities in direct and indirect boundary element schemes (Fukui 1986, Tian 1990, Mack & Crouch 1991, Koller et al. 1992, Loken 1992, Andrews 1994, Siebrits & Crouch 1994), although some hint at the possibility (e.g. Mansur 1983, Antes 1985, Manolis et al. 1986). One of the reasons for the lack of evidence of numerical instabilities in the past has been the very long computer run times and large disk storage requirements of dynamic boundary element methods, especially those using numerical spatial integrations. With the advent of more powerful computers, and the development of boundary element methods using analytical integrations in space and time, the numerical instability issue has become more noticeable.

The time-marching direct boundary element formulations presented by Mansur (1983) and Antes (1985) for two-dimensional problems illustrate increased oscillations at late times, indicating potential instabilities in some of their results. Fukui (1986) notes that his numerical scheme is unstable for smaller time increments. Manolis et al. (1986) present a three-dimensional time domain direct boundary element method, with results for less than 25 time steps. This scheme uses repeated averaging (in time) 'for added accuracy' of the method, and thus hints at the possibility of numerical problems. Mack & Crouch (1991) present a time-marching three-dimensional displacement discontinuity method, and show clear evidence of the development of numerical instabilities at late times. Also, Tian (1990) and Loken (1992) note numerical instabilities in their two- and three-dimensional fictitious stress methods.

Koller et al. (1992) present a regularized (i.e. in which the singularity is weaker than the displacement discontinuity method) boundary integral approach for modeling antiplane crack problems, and show evidence of numerical instabilities. They hypothesize that these instabilities, which deteriorate with more elements, are a result of a spatial instability (Peirce & Siebrits (1996) have shown that numerical instabilities in a model problem are a function of both the time and space discretizations).

Andrews (1994) presents a time domain boundary integral scheme, based on the Green's function approach of Das & Aki (1977), for modeling two-dimensional fracture propagation. He finds that this scheme becomes unstable after approximately 200 time steps for his particular problem. He also investigates a Fourier domain approach for the spatial convolutions, and finds this scheme to become unstable within 150 time steps. He concludes that, were he to redo his work, he would use a finite difference method.

Siebrits & Crouch (1994) present a two-dimensional time-marching displacement discontinuity method, using analytical integrations in both time and space. They show that higher order spatial interpolations reduce the problem of numerical instabilities, but do not eliminate it.

Few papers have been published which examine the numerical instability problem in detail, and try to find solutions to it. Cole et al. (1978) present an analysis of the phenomenon of numerical instabilities, evident in their antiplane strain boundary integral scheme. Banerjee et al. (1986) suggest that implementation errors are responsible for numerical problems in Cole et al.'s work.

Peirce & Siebrits (1996, 1997) have shown, using model problem studies, that numerical stability depends on the type of discretization used in both space and time. If the stress, remote from a source element, is larger than the effect of the source element on itself, then positive feedback will generate an instability. The authors have developed mathematical routines, based on the detection of the roots of a characteristic determinant of the problem, which can detect a priori whether a particular boundary element problem will become unstable.

Birgisson (1996) notes that the introduction of a characteristic length, such as a layer thickness, into a problem changes the nature of energy propagation in elastodynamic problems significantly. Any surface or interface waves that are generated become dispersive and thereby distribute the energy associated with the initial disturbance over a number of elements, because waves of different frequencies have different phase velocities and different energy transport velocities. This will lead to a reduction in the amplitudes of disturbances away from the source, when compared to non-dispersive waves, thus resulting in increased stability.

IMPROVED ALGORITHMS

Some authors claim to have developed unconditionally stable implicit schemes (e.g. Ahmad & Banerjee 1988), but the only proof they show of their claims is to present results for 25 time steps. Evidence can easily be found to show that instabilities usually—but not always—become evident after a certain number of time steps, normally greater than 25.

Others claim to have developed conditionally stable schemes. Belytschko & Chang (1988) present a direct boundary element scheme for antiplane problems. This formulation is said to be stable provided that the time step is less than double the inverse of the natural frequency of the system. No proof is supplied, except for a half-plane problem run for 100 time steps.

Wang and Takemiya (1992) present a direct boundary element method for two-dimensional problems, using analytical integrations in space and time. They claim that their method is stable, so long as 'the time step is smaller (but not too small to exceed a computer's capacity) than the critical sizes characterized by the actual problem.' They demonstrate their claim by showing results for 25 time steps. Again, it is noted that unstable behavior usually manifests itself well after 25 time steps. Some results in Wang (1991) do show increased oscillations at later times, which may possibly be associated with instabilities.

To date, there have been a number of effective ways to reduce the problem of numerical instabilities in time-marching boundary element methods. The variational formulation (e.g. Ding et al. 1989) is more stable than the standard direct schemes because of an extra spatial integration, which reduces the order of singularity of the system. The solution of crack problems can also be obtained using a variational approach (e.g. Becache & Ha Duong 1994). The variational method should be tested to prove its stability properties, and systematically compared with other boundary element methods.

The new time-marching algorithms of Peirce & Siebrits (1997), which can be patched into the existing indirect methods, have also proved to be very effective in reducing substantially, and in many cases eliminating, numerical instabilities (Siebrits & Peirce 1997). Siebrits & Peirce (1997) have shown that the so-called 'half-step' time-marching scheme dramatically improves computer run times, accuracy, and stability of the dynamic displacement discontinuity method. Unfortunately, this scheme does not completely eliminate numerical instabilities in this method. It does, however, allow one to model more sophisticated problems than before. Figure 6 is the equivalent result to Figure 2, but obtained using the half-step approach, and run to 3000 time steps. In this particular case, the instability has not reappeared.

Birgisson et al. (1997) have developed a modified version of the half-step scheme to model traction boundary value problems in the time domain direct boundary element method, and the so-called 'double-step' time-marching scheme for displacement boundary value problems. These methods improve the stability properties of the direct methods substantially. Mixed boundary value problems can potentially be solved by a special combination of the half-step and the double-step schemes.

It should be noted that Birgisson et al. (1997) also found that the so-called 'epsilon' scheme, developed originally by Peirce & Siebrits (1997), and trivial to implement, was found to make time domain direct boundary element methods considerably more stable, contrary to what was found for time domain indirect methods. The epsilon scheme was also found to produce a temporal phase shift in the results of the indirect methods, but not in the direct methods. This scheme is therefore a viable option for improving the stability characteristics of direct methods.

In addition, Nardini & Brebbia (1985) and Brebbia (1986) and others have developed the domain or mass matrix approaches, where the elastostatic fundamental solutions are used, and the inertial terms are included in a domain integral. It is not clear how much better these approaches are over the conventional ones in terms of stability and speed, in light of the necessity to collocate throughout the domain.

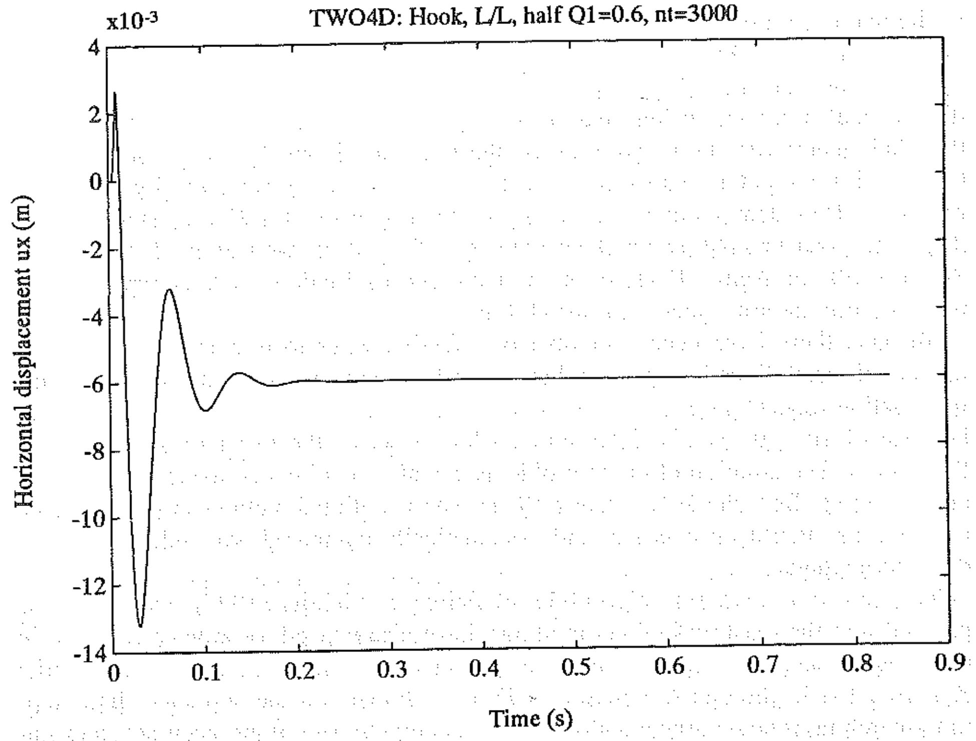

Figure 6. Half-step result to 3000 time steps showing no evidence of an instability.

These methods may also have the drawback of not being able to model sharp loading conditions as effectively as time domain approaches.

A large proportion of reported work on elastodynamic boundary element methods has made use of transform domain approaches, rather than time domain approaches (Beskos 1993). The governing partial differential equations in the time domain approach are hyperbolic, whereas the transform domain approaches reduce the problem to a sequence of standard elliptic partial differential equations, commonly solved in elastostatics. Elliptic equations are more amenable to numerical approximations by the boundary element method than hyperbolic equations (Cruse 1987), which may explain the prevalence of the transform domain approaches in the literature.

Another advantage of the transform domain methods is that material damping is relatively easy to include in the formulation to enhance the numerical stability of the solution. Unfortunately, the use of artificial damping tends to change the amplitudes of the predicted displacements and stresses, which cannot be justified on numerical grounds. It should also be noted that transform domain formulations do not allow one to solve non-linear problems, such as frictional crack behavior, where the solution at each time-step must be solved iteratively, and updated from time-step to time-step.

The development of direct boundary element methods where time is transformed into the Fourier (e.g. Rezayat et al. 1986) or Laplace (e.g. Narayan & Beskos 1982) domains is well documented. The indirect fictitious stress and displacement discontinuity methods in the Laplace domain (Wen et al. 1994) have also been developed. These approaches have been shown to work well, at least for low to moderate frequencies, but they can exhibit numerical problems at higher frequencies (Rezayat et al. 1986, Andrews 1994). This is particularly prevalent in half-space and half-plane problems if use is made of the half-space fundamental solution (e.g. see Rizzo et al. 1985, and Dominguez & Meise 1991), and in the modeling of underground structures with the full-space fundamental solution (Beskos 1993). Chirino et al. (1994) note that spurious oscillations are evident in the Fourier domain direct formulation for purely elastic cases. These oscillations are said to be because of numerical difficulties in the representation of the frequency-response function with zero damping in the discrete Fourier transform.

Mention should also be made of the so-called consistent infinitesimal finite element cell method of Wolf & Song (1996), which can be viewed as a special boundary finite element technique for obtaining a finite element stiffness matrix for an infinite region. In this method, only the boundary is discretized, and the effects of waves radiating into the infinite surroundings is automatically taken care of. The method combines the best of the finite and boundary element methods—the problem dimension is reduced by one, no fundamental solution is required, and the radiation conditions are fully satisfied. This method should also be fully tested on transient problems to check its numerical stability properties.

FUTURE POSSIBILITIES

What is currently needed in the boundary element community is a concerted effort to firstly, candidly recognize that numerical stability problems do arise in both the direct and indirect methods, and secondly, to find better ways of alleviating or even eliminating these problems. If these methods could be improved to the point where numerical stability can be guaranteed a priori, then there would be tremendous scope for the further development of these methods.

More rigorous analysis is needed to examine the effect that the choice of spatial shape function has on the stability of time-marching boundary element methods, especially the displacement discontinuity method, whose fundamental solutions are hyper-singular. Methods should be developed which incorporate displacement continuity at the element edges, preferably without a significant increase in the number of unknowns in the resulting system of equations. Such methods have been shown to improve the accuracy of the elastostatic displacement discontinuity method (Napier 1997, pers. commun.).

In the case of the indirect schemes, multiple interacting crack growth models could be developed very rapidly (equivalent to the static crack growth models of Napier & Peirce 1995) should a complete solution be found to the numerical instability problem. In the case of the direct schemes, far more sophisticated geometric configurations could be routinely modeled. In addition, dynamic boundary element methods could be streamlined by the inclusion of numerical lumping or multipole expansion methods in space and time to dramatically reduce the effective size of the coefficient matrix, and hence the computer run times, with minimal loss in accuracy.

CONCLUSIONS

We have summarized evidence of numerical instabilities in the two- and three-dimensional direct and indirect time domain boundary element methods. We have pointed to improved time-marching algorithms and alternative formulations which are more stable than the conventional approaches. We look forward to seeing a concerted effort by the boundary element community to investigate and find solutions to this problem, so that dynamic boundary element methods can reach a point where they surpass other numerical methods, and can be used to model more intricate problems very accurately without numerical stability problems.

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Blast wave propagation in rock mass—Part I: monolithic medium

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ABSTRACT: This contribution presents an introduction into that part of the field of elastic wave propagation which is relevant to blasting of single and multiple boreholes and addresses briefly borehole breakdown and the formation of the fracture network around a borehole which ultimately leads to the disintegration of the rock mass. Phenomena of this kind are encountered in mining and quarrying. Analytical as well as numerical work referring to stress wave expansion from a detonating borehole will be addressed in detail.

Much of the research reported here has been performed at the Fracture and Photo-Mechanics Laboratory (FPML) at the Vienna University of Technology. In order to gain an improved understanding of the mechanisms controlling borehole breakdown and ensuing rock fragmentation, a multidisciplinary approach is followed which includes analytical work and numerical modeling. The main emphasis is put on the effect of the velocity of detonation on the stress wave propagation and interaction with the rock mass which may contain a variety of special features such as layers, delaminations, faults and other geological structural discontinuities.

This paper consists of two parts, and describes the results of numerical simulations demonstrating the applicability of the dynamic finite difference program SWIFD. The following topics will be discussed in each part: in Part I (this article), blast wave propagation in monolithic rock; in Part II, the interaction between stress waves and geological features of discontinuity such as interfaces, cracks and faults.

KEYWORDS: blasting, velocity of detonation, layered rock, fracture, delamination, fault, discontinuity, wave propagation, Rayleigh wave, borehole breakdown.

1 INTRODUCTION

One of the most important problems in mining and quarrying is the efficient disintegration of the rock mass in order to produce pay material without additional time and money consumed by secondary fragmentation (particularly of large incompletely fractured boulders). In addition, it is important to predict the effect of any stress waves that are generated during the liberation of the energy of the explosive charge on the behavior of the rock mass and on any possible structures embedded in the rock mass or located on the surface.

Any advanced investigation of the physical phenomena occurring in the close vicinity of a borehole during and shortly after the detonation of a cylindrical charge of commercial explosives requires the application of the disciplines of rock fracture mechanics, continuum damage mechanics and wave propagation in solids. Although many of the phenomena and mechanisms triggered by a detonation are fairly well understood from a qualitative point of view, much work remains to be done in quantifying the results.