Problem 1. For the following relations, decide if they are equivalence relations or not and justify your answer.

1. On the set of all lines in the $xy$ plane, we define the relation $\mathcal{R}$ by: $D \mathcal{R} D'$ if $D$ and $D'$ are orthogonal.

2. On the set of all lines in the $xy$ plane, we define the relation $\mathcal{R}$ by: $D \mathcal{R} D'$ if $D$ and $D'$ are parallel (possibly $D = D'$).

3. On $\mathbb{R}$, we define the relation $\mathcal{R}$ by: $x \mathcal{R} y$ if $\cos^2(x) + \sin^2(y) = 1$.

4. On $X = \{(a, b), a \in \mathbb{Z}, b \in \mathbb{N}\}$, we define the relation $\mathcal{R}$ by: $(a, b) \mathcal{R} (a', b')$ if $ab' = a'b$.

5. On $\mathbb{R}$, we define the relation $\mathcal{R}$ by: $x \mathcal{R} y$ if $|x - y| < 1$.

Problem 2. Let $A = \{1, 2, 3, 4, 5, 6\}$. Write out the relation $\mathcal{R}$ given by $x \mathcal{R} y$ when $x \nmid y$ (where $\nmid$ means "does not divide") on $A$ as a set of ordered pairs (ie write $\mathcal{R}$ as a subset of $A \times A$).

Problem 3. Let $\mathcal{R}$ be a symmetric and transitive relation on a set $A$. (These assumptions apply to both parts 1. and 2. of this problem.)

1. Show that $\mathcal{R}$ is not necessarily reflexive.

2. Suppose that for every $a \in A$, there exists $b \in A$ such that $a \mathcal{R} b$. Prove that $\mathcal{R}$ is reflexive.

Given an equivalence relation $\mathcal{R}$ on a set $A$ and $a \in A$ we define the equivalence class of $a$ by

$[a]_\mathcal{R} = \{b \in A, a \mathcal{R} b\}$.

It is a subset of $A$. The set of all equivalence classes is often denoted by $A/\mathcal{R}$. We have $A/\mathcal{R} \subseteq \mathcal{P}(A)$ (namely an element in $A/\mathcal{R}$ is a subset of $A$).

Problem 4. We consider the relation $\mathcal{R}$ defined on $A = \{x \in \mathbb{Z}, |x| \leq 8\}$ by $x \mathcal{R} y$ if $x^2 \equiv y^2 \mod 6$.

1. Check that $\mathcal{R}$ is an equivalence relation (not to be handed in).

2. Give the list of all equivalence classes.

Problem 5. Let $\mathcal{R}$ be the relation defined on $\mathbb{R} \times \mathbb{R}$ by

$(x_1, y_1) \mathcal{R} (x_2, y_2)$ if $x_1^2 + y_1^2 = x_2^2 + y_2^2$.

1. Check that it is an equivalence relation (not to be handed in).

2. Describe the equivalence classes. You can make a drawing.