Problem 1. We consider subsets $A$, $B$ and $C$ of the universe $\Omega$.

1. Prove that $\overline{A} \subseteq B$ if and only if $A \cup B = \Omega$.
2. Prove that $\overline{A} \subseteq B$ implies $(C \setminus B) \cup A = A$

Problem 2. Let $a \in \mathbb{R}$.

1. On the $xy$-plane, draw the set $A_a = \{(x, x^2 - ax), x \in \mathbb{R}\}$ when $a = 0$, $a = 1$ and $a = 2$.
2. Show that $\bigcap_{a \in \mathbb{R}} A_a = \{(0, 0)\}$.

Problem 3. Chapter 8 Problem 28. (You can use ideas from HW5 Problem 4).

Problem 4. Prove
\[ \bigcup_{n \in \mathbb{N}} \left[ \frac{1}{n}, 1 \right] = (0, 1). \]

We admit: for every $x \in \mathbb{R}$, there exists a unique $m \in \mathbb{Z}$ such that $x \in [m, m+1)$. See HW5 Problem 6.

Problem 5. Let $A$, $B$ and $C$ be sets. For each of the following statements, either prove it is true or give a counterexample.

1. $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$
2. $\mathcal{P}(A \cup B) \supseteq \mathcal{P}(A) \cup \mathcal{P}(B)$
3. $\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B)$
4. $\mathcal{P}(A \cap B) \supseteq \mathcal{P}(A) \cap \mathcal{P}(B)$
5. $(A \cup B) \setminus C) \cup (A \cap B \cap C) \subseteq (A \setminus (B \cup C)) \cup (B \setminus (A \cup C))$.

Problem 6. For the following relations, decide if they are equivalence relations or not and justify your answer.

1. On the set of all lines in the $xy$ plane, we define the relation $R$ by: $D \sim D'$ if $D$ and $D'$ are orthogonal.
2. On $\mathbb{R}$, we define the relation $R$ by: $x \sim y$ if $\cos^2(x) + \sin^2(y) = 1$.

Problem 7. Let $\mathcal{R}$ be the relation defined on $\mathbb{R} \times \mathbb{R}$ by
\[ (x_1, y_1) \mathcal{R} (x_2, y_2) \text{ if } x_1^2 + y_1^2 = x_2^2 + y_2^2. \]

1. Check that it is an equivalence relation.
2. Describe the equivalence classes. You can make a drawing.

Problem 8. Let $\mathcal{D}$ denote the set of all lines in the $xy$-plane which are parallel (or identical) to the line with equation $y = 3x$. Each such line is treated as a subset of $\mathbb{R} \times \mathbb{R}$.

1. Check for yourself that $\mathcal{D}$ is a partition of $\mathbb{R} \times \mathbb{R}$. (Not be to handed in).
2. Give an equivalence relation $\mathcal{R}$ on $\mathbb{R} \times \mathbb{R}$ whose set of equivalence classes is $\mathcal{D}$.