Problem 1. Consider three statements $P$, $Q$, $R$ statements.

1. Using truth tables, recall the proof of the logical equivalence between

$$P \land (Q \lor R) \text{ and } (P \land Q) \lor (P \land R).$$

This is one of the distributivity laws.

2. Deduce the other distributivity law namely the logical equivalence between

$$P \lor (Q \land R) \text{ and } (P \lor Q) \land (P \lor R).$$

without using a truth table but instead using DeMorgan’s laws.

3. Let $P$, $Q$, $R$ and $S$ be statements. Suppose that

$$P \text{ is false and } (R \Rightarrow S) \iff (P \land Q) \text{ is true.}$$

Find the truth values of $R$ and $S$.

Problem 2. Prove the following statement:

Let $a$ and $b$ be integers. If $(a - 2)^2(ab + 2)$ is odd, then $a$ and $b$ are odd.

Problem 3. 1. Prove the following statement. For every $a \in \mathbb{R}$.

if $a \geq 4$, then $-\frac{a^2}{4} + a \leq 0$. 

2. Let $a \in \mathbb{R}$. Prove the following statement:

$$(x^2 + ax + a > 0 \text{ for every } x \in \mathbb{R}) \text{ if and only if } (0 < a < 4).$$

You may want to transform the expression $x^2 + ax + a$ by completing the square.

Problem 4. Write the negation of the following statements. Avoid using ”there is no”, ”is not” as much as possible. You may use the sign $\neq$.

1. There is a problem that has no solution.

2. Every house in this city has at least 4 windows.

3. For any $x \in \mathbb{R}$, if $x > 10$ then $2^{-x} < 1$.

4. For every $z \in \mathbb{Z}$, there exist numbers $x \in \mathbb{Z}$ and $y \in \mathbb{Z}$ such that $z^2 = x^2 + y^2$.

Problem 5. (Chapter 4 Exercise 8). Suppose $a$ is an integer. Prove the following statement:

if $5|2a$, then $5|a$. 