MATH 419/545
HW 5

Due March 20, 2020 (at the beginning of the class)

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1. Exercise 5.3.7 (Here p is irreducible means that S is irreducible. Equivalently $[i] = S$ for all $i \in S$.)

2. Exercise 5.3.5

3. Let p be a transition function on the countable set S and $i \in S$. Assume for each $j \in S$,
$$\lim_{n\to\infty} p^n(i,j) = \pi(j),$$
where π is a probability measure on S.

   (a) Prove that for any bounded $g : S \to \mathbb{R}$, $\lim_{n\to\infty} E_i(g(X_n)) = \int_S g\, d\pi$.

   (b) Prove that π is a stationary distribution for p.

4. Given an example of a stochastic process $\{X_n : n \in \mathbb{Z}_+\}$ on $(\Omega, \mathcal{F}, P)$ such that $P_{X_n} = P_{X_0}$ for all n but $\{X_n\}$ is not stationary. Here P_{X_n} denotes the pushforward measure of P under X_n.

5. Assume $\{Y_n : n \in \mathbb{Z}_+\}$ are iid with $P(Y_0 = 0) = P(Y_0 = 1) = 0.5$. Let $X_n = Y_n + Y_{n+1}$ for all $n \in \mathbb{Z}_+$.

   (a) Prove that X is not a $(\mathcal{F}_n^X)$-Markov chain by showing that $P(X_2 = 2|\mathcal{F}_1^X)$ is not equal to $\phi(X_1)$ for any $\phi : \{0,1,2\} \to [0,1]$.

   (b) Prove that X is a stationary process.

Practice Problems (do not hand in)

1. For any state $i \in S$, $[i]$ is irreducible. Further if i is recurrent, then $[i]$ is closed.

2. If $i, j \in S$, then either $[i] = [j]$ or $[i] \cap [j] = \emptyset$.

3. Exercise 5.3.1

4. Exercise 5.3.2

5. Exercise 5.3.4