Math 105 Assignment 3
Due the week of January 24

1. Calculate the definite integrals $\int_0^{\frac{\pi}{4}} \tan^2\theta \sec^3\theta\, d\theta$. (4 pts)

$$J = \int \tan^2\theta \sec^3\theta \, d\theta = \int (\sec^2\theta - 1)\sec^3\theta\, d\theta = \int \sec^5\theta\, d\theta - \int \sec^3\theta\, d\theta = I_5 - I_3$$

$$I_5 = \int \sec^5\theta\, d\theta = \int \sec^3\theta \cdot \sec^2\theta\, d\theta$$

$$= \sec^3\theta \tan\theta - \int 3\sec^2\theta \cdot \sec\theta\tan\theta \cdot \tan\theta\, d\theta \quad \left\{ \begin{array}{l} \text{using integration} \\ \text{by parts with} \\ u = \sec^3\theta,\ v' = \sec^2\theta. \end{array} \right.$$

$$= \sec^3\theta \tan\theta - 3\int \sec^3\theta \cdot (\sec^2\theta - 1)\, d\theta$$

$$= \sec^3\theta \tan\theta - 3I_5 + 3I_3$$

so $\boxed{4I_5 = \sec^3\theta \tan\theta + 3I_3}$. — (1)

similarly, $I_3 = \int \sec^3\theta\, d\theta = \int \sec\theta \sec^2\theta\, d\theta$

$$= \sec\theta\tan\theta - \int \sec\theta\tan\theta\tan\theta\, d\theta \quad \left\{ \begin{array}{l} \text{using integration} \\ \text{by parts again with} \\ u = \sec\theta,\ v' = \sec^2\theta \end{array} \right.$$

$$= \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1)\, d\theta$$

$$= \sec\theta\tan\theta - I_3 + \int \sec\theta\, d\theta$$

so $\boxed{2I_3 = \sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|}$ — (2)

Combining (1) & (2) we get: $J = I_5 - I_3 = \frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{4}I_3 - I_3$

$$= \frac{1}{4}\sec^3\theta\tan\theta - \frac{1}{4}\cdot\frac{1}{2}\left[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right]$$

$$\therefore \int_0^{\pi/4} \tan^2\theta \sec^3\theta\, d\theta = \frac{1}{4}\cdot 2\sqrt{2} - \frac{1}{8}\left(\sqrt{2} + \ln(\sqrt{2}+1)\right) = \boxed{\frac{3}{8}\sqrt{2} - \frac{1}{8}\ln(\sqrt{2}+1)}$$