

Mock Midterm

MATH 105 Section 208 (MacLean)

March 4, 2010

1. A manufacturer is planning to sell a new product at \$150 per unit, and estimates that if it spends x thousand dollars on development and y thousand dollars on promotion, then it will sell

$$q = q(x, y) = \frac{320y}{y+2} + \frac{160x}{x+4}$$

units of product. The cost of manufacturing the product is \$50 per unit. The manufacturer has a total of \$8000 to spend on development and promotion. Use the method of Lagrange multipliers to find how this money should be allocated to generate the largest possible **profit**. [*Hint*: Profit = (number of units sold)(price per unit - cost per unit) - (amount spent on development and promotion).]

2. Find the area of the finite region bounded by the x -axis, the curve $y = x$, and the curve $y = 2/(x+1)$. It will be useful to sketch the region before attempting the calculation.
3. Suppose that the marginal revenue function for a company producing q units of a product is

$$MR(q) = R'(q) = 400 - 3q^2.$$

Find the *additional* revenue received from doubling production if currently 10 units are being produced.

4. A tire manufacturer estimates that q thousand radial tires will be demanded by wholesalers when the price is

$$p = D(q) = -0.1q^2 + 90$$

dollars per tire, and that the same number of tires will be supplied when the price is

$$p = S(q) = 0.2q^2 + q + 50$$

dollars per tire.

- (a) Find the equilibrium price (i.e. where the supply and demand curves intersect) and the quantity supplied and demanded at that price. A sketch may be useful.
 - (b) Determine the consumers' and the producers' surpluses at the equilibrium price.
5. An oil well that yields 900 barrels of crude oil per month will run dry in 3 years. The price of the crude oil is currently \$40 per barrel. If the oil is sold as soon as it is extracted from the ground and the money invested at 5% per year compounded continuously, what will the total future value of the revenue from the well over its 3-year lifetime?