Q1 [5 marks]

$\int_1^2 \ln x \, dx =$

Let $u = \ln x \quad dv = dx$

$\Rightarrow du = \frac{1}{x} \quad v = x$

$$\int_1^2 \ln x \, dx = x \ln x \Big|_1^2 - \int_1^2 x \cdot \frac{1}{x} dx$$

$$= 2 \ln 2 - 1 \ln 1 - x \Big|_1^2$$

$$= 2 \ln 2 - 2 + 1 = \boxed{2 \ln 2 - 1}$$

Q2 [5 marks]

$\int x^2 e^x \, dx =$

Let $u = x^2 \quad dv = e^x dx$

$du = 2x \, dx \quad v = e^x$

$$= x^2 e^x - \int 2x e^x dx$$

Let $u = 2x \quad dV = e^x dx$

$\Rightarrow du = 2dx \quad V = e^x$

$$= x^2 e^x - 2x e^x + \int 2 e^x dx$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$= \boxed{(x^2 - 2x + 2) e^x + C}$$

Q3 [5 marks]

$\int \cos x \ln(\sin x)\, dx =$

Let $u = \sin x$

$du = \cos x\, dx$

$= \int \ln u\, du$

$= u \ln u - u + C$

$= \boxed{\sin x \ln(\sin x) - \sin x + C}$

Q4 [5 marks]

$\int_0^3 \frac{v^2+1}{\sqrt{v^3+3v+4}}\, dv =$

$v = 0 \longmapsto u = 4$

$v = 3 \longmapsto u = 40$

Let $u = v^3 + 3v + 4$

$\Rightarrow du = 3v^2 + 3\, dv = 3(v^2+1)dv$

$$\frac{1}{3}\int_4^{40} \frac{du}{\sqrt{u}} = \frac{1}{3} \cdot \frac{u^{1/2}}{1/2}\Big|_4^{40}$$

$$= \frac{2}{3}\left(\sqrt{40} - \sqrt{4}\right) = \frac{2}{3}\left(2\sqrt{10} - 2\right)$$

$$= \boxed{\frac{4}{3}\left(\sqrt{10} - 1\right)}$$

Q5 [5 marks]

$\int (x^{3/2}+8)^5 \sqrt{x}\, dx =$

Let $u = x^{3/2} + 8$

$du = \frac{3}{2} x^{1/2} dx$

$= \int \frac{2}{3} u^5 du$

$= \frac{2}{3} \frac{u^6}{6} + C$

$= \boxed{\frac{1}{9}(x^{3/2}+8)^6 + C}$