Last Name: _____   **First Name:** _____

Student Number: _____

[10]   **1.**   Consider the function
$$f(x, y) = xye^{(-x^2+y^2)/2}.$$

.

(a)   Find all the critical points of $f(x, y)$.

(b)   Use the second derivative test to classify the critical points you found in part (a).

[10] **2.**   Use the method of Lagrange multipliers to find the maximum and minimum values of

$$f(x, y) = x^2 + 4xy$$

subject to the constraint

$$x^2 + y^2 = 1.$$

[10] **3.**   The Cheapo Company makes two products, A and B, that sell for $10 and $9 per unit, respectively. The cost of producing x units of A and y units of B is given by

$$C(x, y) = 400 + 2x + 3y + 0.01(3x^2 + xy + 3y^2).$$

Find the values of x and y that maximize *profit*.

[10] **4.** Suppose you have a production function $P = f(K, L) = K^a L^{1-a}$, where $0 \leq a \leq 1$, and K is the capital available and L is the labour available. Suppose that you have a budget constraint $pK + qL = B$, where p, q, and B are given positive constants. Show that the production P is maximized when $K = aB/p$ and $L = (1 - a)B/q$.

The End