More on related rates. §3.11

Ex 1. A spherical balloon is being inflated at a rate of 13 cm³/sec. How fast is the radius changing when the balloon has radius 15 cm?

[Diagram of a sphere]

Volume of sphere:

\[ V(t) = \frac{4}{3} \pi r^3 \]

WANT to find \( \frac{dr}{dt} \) when \( r = 15 \) cm.

Differentiate: \( \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \).

We know \( \frac{dV}{dt} = 13 \text{ cm}^3/\text{sec} \).
plug in. numbers.

\[ 13 = 4\pi \cdot (15)^2 \frac{dr}{dt} \]

\[ \frac{dr}{dt} = \frac{13}{4\pi \cdot 15^2} \]

\[ = \frac{13}{4\pi \cdot 225} \]

\[ = \frac{13}{900\pi} \text{ cm/sec} \]

P22b 6x4

An observer stands 200 meters from the launch site of a hot-air balloon. The balloon rises vertically at a constant rate 4 m/s. How fast the angle of elevation of the balloon increases 30 seconds after launch.
WANT to find \( \frac{d\theta}{dt} \) when \( t = 3a \) sec.

We know \( \frac{dy}{dt} = 4 \text{ m/s} \).

\( \theta \): angle of elevation

\[
\tan \theta(t) = \frac{Y(t)}{Zo} \tag{1}
\]

When \( t = 30 \text{ sec} \), \( Y(t) = 4 \text{ m/s} \cdot 30s. \)

\[
= 120 \text{ m}.
\]

Differentiate (1):

\[
\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{200} \cdot \frac{dy}{dt}
\]

\[
\sec^2 \theta = \frac{1}{\cos^2 \theta}
\]

\[
\frac{d\theta}{dt} = \frac{1}{200} \cdot 4 \cdot \frac{1}{\sec^2 \theta}
\]

\[
= \frac{1}{50} \cos^2 \theta
\]
\[
\cos \theta = \frac{200}{\sqrt{200^2 + 120^2}} \approx 0.86
\]

Plug in \( \cos \theta \).

\[
\frac{dB}{dt} = \frac{1}{50} \cdot 0.86^2 \text{ rad/sec.}
\]
§4.1 Maxima & Minima.

Definition.

A function \( f \) is defined on a set \( D \).

If \( f(c) \geq f(x) \) for every \( x \) in the set \( D \), \( f(c) \) is an absolute maximum.

If \( f(c) \leq f(x) \) for every \( x \) in \( D \), \( f(c) \) is an absolute minimum.

Ex. 2. \( y = x^2 \) on \([0, 2]\)
Ex 2.

\[ y = x^2 \] on \((-\infty, \infty)\)

abs min is \(0\).
No absolute maximum.

Ex 3.

\[ y = x^2 \] on \((0, 2)\)

No absolute maximum nor absolute minimum.
Question. Does a function \( f \) always have abs. max. or min. on a closed interval \([a, b]\)?

\[
\begin{align*}
 f(x) &= \begin{cases} 
 0 & 0 < x < 1 \\
 1 & x = 1 
\end{cases} \\
 \text{not continuous on } [0, 1] \\
\text{no absolute max.} \\
\text{absolute min is 0.}
\end{align*}
\]

\textbf{Thm. Extreme Value Theorem.}

A function that is \textit{continuous} on a closed \([a, b]\) has an absolute maximum value and absolute minimum value on the interval.