\[ f'(x) = \frac{\tan x}{\sec^2 x} \left( \frac{10 \sec^2 x}{(10 \sec x - \frac{3\pi}{5})} \right) \]

\[ f'(x) = \frac{1}{\sec x} \cdot \frac{10}{\sec x} \cdot \frac{\sec x}{(10 \sec x - \frac{3\pi}{5})} \]

\[ \frac{d}{dx} \ln(\tan x) = \frac{1}{\tan x} \]

\[ \frac{d}{dx} \left( \frac{10}{(5x+3)} \right) \]

\[ \ln(\tan x) = \frac{\tan x}{(5x+3)} \]

\[ \ln(\tan x) = \frac{\sec^2 x}{(5x+3)} \]

\[ \ln(\tan x) = \frac{\tan x}{(5x+3)} \]
Price elasticity of demand function $R(p)$.

- Revenue: $R(p) = \text{price} \times \text{quantity}$

1. \[\text{price} = P \cdot q(p)\]

2. \[R'(p) = 1 \cdot q(p) + P \cdot \frac{dq}{dp} \cdot \frac{q(p)}{q(p)} = q(p) \left(1 + \frac{P}{q} \cdot \frac{dq}{dp}\right)\]

3. \[\varepsilon(p) = \left| \frac{\frac{P}{q} \cdot \frac{dq}{dp}}{1} \right| \leq 0.\]

Demand elasticity

- $q(p) > 0$
- $1 + \varepsilon < 0$
- $R'(p) < 0$. Revenue decrease when price increase.

[Diagram indicating the direction of $\varepsilon(p)$ determined by the negative sign of $\frac{dq}{dp}$]
$1 + \varepsilon < 0$. $R'(p) < 0$. Revenue decreases when increasing the price.

$1 + \varepsilon > 0$. $R'(p) > 0$. ... increases when increasing the price.

$\varepsilon = -1$. $R'(p) = 0$. Has the maximum revenue.
\[ \frac{\partial Q}{\partial P} < 0 \quad \text{price elastic} \]

\[ \frac{\partial Q}{\partial P} > 0 \quad \text{price inelastic} \]

\[ \frac{\partial^2 Q}{\partial P^2} = \frac{\text{percentage change in demand}}{\text{percentage change in price}} \]
Ex 2 Suppose the demand curve is given by

\[ q = 500 - 10p. \]

(a) Price elasticity?

\[ E(p) = \frac{P}{q} \frac{dq}{dp} = \frac{P}{q} \cdot (-10) \]

\[ = \frac{P}{500 - 10p} \cdot -10 = \frac{-10P}{500 - 10p}. \]

\[ = - \frac{P}{50 - p} \]
(b) price elasticity when $P = 30$

$$E(30) = -\frac{30}{50-30} = -\frac{3}{2}$$

$$\left|\frac{3}{2}\right| > 1$$  price elastic

We should consider lowering price to raise revenue.

"percentage change in demand if the price is $30$ and increased by 4.5%"

$$4.5\% \times -\frac{3}{2} = -6.75\%$$

means demand decreased by 6.75%
Ex2. demand and price relations is.

\[ 9p + 30q + 50q = 8500. \]

Question: At price $150, will revenue increase or decrease by increasing price when is the revenue maximized.

1. Find \( E(p) \), evaluate \( E(150) \) and compare with 1.
$$S(P) = \frac{P}{q} \frac{dq}{dp}.$$ 

Use implicit differentiation.

$$\frac{dq}{dp} \cdot P + q + 30 + 50 \frac{dq}{dp} = 0$$

$$(p+50) \frac{dq}{dp} + q + 30 = 0$$

$$\frac{dq}{dp} = -\frac{(q+30)}{p+50}$$

3. (150) =

when $P = 150$

$$150q + 4500 + 50q = 5500$$

$q = 20.$