Average and Marginal Cost.

The cost function \( C(q) \).

The average cost \( \bar{C}(q) = \frac{C(q)}{q} \).

The marginal cost is \( \frac{dc}{dq} \).

Means the approximate cost to produce one additional item after \( q \) item.
Growth model.

A measure of a quantity (population, price index ...) at time \( t \).

\[
P_t \uparrow
\]

Average growth rate between \( [a, \, \text{ath}] \)

\[
R_{avg} = \frac{P(\text{ath}) - P(a)}{\text{ath} - a}
\]

Instantaneous growth rate

\[
\Gamma_{inst}(a) = P'(a) = \left. \frac{dP}{dt} \right|_{t=a}
\]
(c) When is the profit maximized?

\[ c'(q) = P'(q) \]

\[ 75 = 300 - \frac{1}{25}q \]

\[ \frac{1}{25}q = 300 - 75 \]

\[ \frac{1}{25}q = 225 \]

\[ q = 5625 \]
So profit is maximised when Marginal Revenue = Marginal Cost.

Q: What is Marginal Cost?

\( \frac{dC}{dq} = 75 \)

\( C(q) = 100000 + 75q \)

\( C'(q) = 75 \)

(b). \( R(q) = 300q - \frac{1}{50}q^2 \)

\( R'(q) = 300 - \frac{1}{50} \times 2q \quad 300 - \frac{1}{25}q \)
Revenue function \( R(q) = q \cdot P(q) \).

Marginal Revenue. \( \frac{dR}{dq} \).

Profit function. \( P(q) = R(q) - (q) \).

Marginal Profit. \( P'(q) = \frac{dP}{dq} \).

Profit function is maximal when \( P'(q) = 0 \).

\( R(q) - C'(q) = 0 \)

If \( R(q) = C'(q) \).

We have maximal profit.
Ex. Bacteria numbers in an experiment are given by

\[ P(t) = e^{0.1t} \]

Q: What is the growth rate of the bacteria?

\[ R_{\text{ins}} = P'(t) = \frac{d}{dt}(e^{0.1t}) \]
Recall. \((e^t)' = e^t.\)
\((e^{kt})' = ke^{kt}.\)
\[e^{0.1t}.\]
\[
\frac{d}{dt} (e^{0.1t}) = 0.1 e^{0.1t}.\]

Growth rate at \(t = 1\) ?
\[p(1) = 0.1 e^{0.1}.\]
Review problem.

1. \[ \lim_{x \to -1} \frac{(2x-1)^2 - 9}{x+1} \]

\[ = \lim_{x \to -1} \frac{4x^2 + 4x - 9}{x+1} \]

\[ = \lim_{x \to -1} \frac{4x^2 - 4x - 8}{x+1} = \]

\[ = \lim_{x \to -1} \frac{4(x^2 - x - 2)}{x+1} \]

\[ = 4 \lim_{x \to -1} \frac{x^2 - x - 2}{x+1} \]

\[ = 4 \lim_{x \to -1} \frac{(x+1)(x-2)}{x+1} \]

\[ = 4 \lim_{x \to -1} (x-2) = -12. \]
2. Explain why 
\[ nx^4 + 25x^3 + 10 = 5 \]
has a solution on \((0, 1)\).

Recall IVT.

\( f(x) \) continuous on 
\([a, b]\), \( f(a) \leq L \leq f(b) \)

there exists \( a < c < b \)

such that \( f(c) = L \).

do we have \( \frac{1}{a} \) such that
\( f(10) = \sqrt{10} - 5 < 0 \)
\( f(1) = \sqrt{1 + 25 + 10} - 5 = 1.70 \)

Let \( f(x) = \sqrt{x^4 + 25x^3 + 10} - 5 \) x on \((0, 1)\) \( f'(x) = 0 \)
fix) is continuous on $(0,1)$
By IVT. there is $c$ on $(0,1)$
satisfying $f(c) = 0$.

$\Rightarrow \sqrt{c^4 + 25c^3 + 10} = 5$.

equivalent