Ex: Find the volume of the solid obtained by revolving the area bounded by \( y = x^2 \), the x-axis, \( x=1 \), and \( x=5 \) about the line \( y = -1 \).

"slice at \( x \)"

radius of large circle = \( 1+x^2 \)
radius of smaller circle = 1

area of the slice at \( x \) = \( \pi \left( (1+x^2)^2 - (1)^2 \right) \)

So the volume of the solid is

\[
\int_{1}^{5} (\text{Area}) \, dx = \pi \int_{1}^{5} (1+x^2)^2 - 1 \, dx.
\]

\[
= \pi \int_{1}^{5} 1 + 2x^2 + x^4 - 1 \, dx = \pi \int_{1}^{5} 2x^2 + x^4 \, dx.
\]

Finish at home.
Ex: Find the volume of the solid obtained by revolving the region bounded by \( y = \sqrt{x}, \quad x = 0, \quad x = 4 \) and the x-axis about the y-axis.

Slice at \( y \):

Radius of the larger circle = 4
Radius of the smaller circle = \( y^2 \).

Area of the "slice at \( y \)":

\[
\pi (4)^2 - \pi (y^2)^2 = \pi (16 - y^4)
\]

Volume = \( \int_{0}^{2} (16 - y^4) \, dy \). (Finish this at home)
Ex: Find the volume of the solid obtained by revolving the region bounded by $y=x$ and $y=\sqrt{x}$ about $x=2$.

"Slice at $y$":

Area at $y$: $\pi (2-y^2)^2 - \pi (2-y)^2$

Volume: $\pi \int_0^1 (2-y^2)^2 - (2-y)^2 \, dy$. 

radius of larger circle = $2 - y^2$

radius of smaller circle = $2 - y$
1.7: Integration by Parts

- Helps us integrate products of functions, e.g. \( \int x e^x \, dx \).

**What is it?** Start w/ product rule:

\[
\frac{d}{dx}(u(x)v(x)) = u'(x)v(x) + u(x)v'(x).
\]

\[\Rightarrow \int u'(x)v(x) + u(x)v'(x) \, dx = u(x)v(x) + C\]

Split apart the LHS and rearrange:

\[
\int u(x)v'(x) \, dx = u(x)v(x) - \int u'(x)v(x) \, dx
\]

If the second integral is easier than the first, we've made progress.

**Ex:** \( \int x e^x \, dx = xe^x - \int e^x \, dx = xe^x - e^x + C \).

\( u(x) = x \quad u'(x) = 1 \)

\( v'(x) = e^x \quad v(x) = e^x \)