

Math 100:V02, Winter Term 2024
Worksheet 16

March 26th, 2024

Instructions

- Find all errors

1. Differentiate $f(x) = 3x^3 + \frac{7}{x^{3/2}}$.

$$f'(x) = 3x^2 + \frac{1 \cdot x^{3/2} - \frac{1}{2} x^{1/2}}{(x^{3/2})^2}$$

2. Evaluate $\lim_{t \rightarrow 3} \frac{(t-3) \sin t}{t^2 - 9}$

$$\frac{(t-3) \sin t}{t^2 - 9} = \frac{0}{0} = \infty$$

3. Determine the asymptotics as $x \rightarrow \infty$ of $f(x) = \frac{\sqrt{x^4 + ax - 1}}{bx + c}$ if a, b, c are nonzero constants.

$$\frac{\sqrt{x^4 + ax - 1}}{bx + c} \sim \frac{ax - 1}{bx} \sim \frac{1}{bx}$$

4. Differentiate $g(x) = \frac{e^{\sin x}}{x^2}$.

$$g'(x) = -\frac{e^{\sin x}}{x^3} + \frac{e^{\sin x}}{x^2}$$

5. Approximate $\sqrt[3]{7}$ using a linear approximation.

Let $h(x) = x^{1/3}$. Then $h'(x) = x^{-2/3}$ so $h'(8) = 8^{-2/3} = 64^{-1/3} = 2^{-1/3}$
 so $h(7) \approx h(8) + 2^{-1/3} \cdot (8 - 7) = 2 \cdot 2^{-1/3} = 2^{2/3}$

6. Find the line tangent to the curve $3x^2y + y^3 = (x + y)^2$ at the point $(1, 1)$.

$$3x^2y' + 6xy + 3y^2 = 2(x + y)$$

so $3y' + 6 + 3 = 4$ so $y' = -2$ so the line is $y = -2(x - 1) + 1$

7. Differentiate the function $x \log x$ with respect to x .

$$(x \log x)' = \log x + \frac{x}{\log x}$$

8. Let f be a function such that $f'(x) = \frac{(x-3)(x+5)}{x^4+1}$. Find the regions where f is increasing and decreasing.

Increase $(3, \infty)$

Decrease $(-\infty, -3)$

9. The volume V of an expanding spherical balloon of radius r is given by $V = \frac{4}{3}\pi r^3$. At the moment when $r = 3\text{cm}$ we have $\frac{dr}{dt} = \frac{1}{\pi} \frac{\text{cm}}{\text{sec}}$. How fast is the volume changing at that moment?

$$V' = 4\pi r^2 = 36\pi$$

10. Find the second order Taylor polynomial of $f(x) = e^{x^2} + x^3$ about $x = 0$.

$$f'(x) = 2xe^{x^2} + x^2$$

$$f''(x) = 2e^{x^2} + 4x^2e^{x^2} + 2x$$

$$T_2(x) = (e^{x^2} + x^3) + (2xe^{x^2} + x^2) \cdot x + \frac{1}{2}(2e^{x^2} + 4x^2e^{x^2} + 2x)$$

11. Suppose the function f has $f(x) \approx 5 + 2(x-3) + (x-3)^3$ to third order about $x = 3$. What is $f''(3)$?

$$f''(x) = 6(x-3)$$

12. Find $\lim_{x \rightarrow 1} \frac{\tan x}{(x-1)^2}$

$$\frac{\tan 1}{0}$$

13. Suppose that $f'(3) = 8$. Find the derivative of $f(x^2 + 2)$ at $x = 1$.

$$f'(1+2) = 8$$

14. Differentiate $\frac{x^2}{x+a}$

$$\frac{2x}{(x+a)^2}$$

15. Determine the asymptotics of $g(x) = \frac{x^7 + 5\sin x + e^{3x}}{x^5 + 3}$ as $x \rightarrow \infty$.

$$\frac{x^7 + 5\sin x + e^{3x}}{x^5 + 3} \sim \frac{x^7}{x^5} = \infty$$

16. Find the fourth order Taylor polynomial of $f(x) = \frac{e^{x^2}}{1+x^2}$ about $x = 0$.

$$e^{x^2} \sim 1 + x^2 + \frac{x^4}{2}$$

$$\frac{1}{1+x^2} \sim 1 - x^2 + x^4$$