Math 100, lecture 25, 11/4/2024

Last time: Multivariable optimization

Fix f of 2, 3, .. variables (say two)

(1) critical points occur over (x,y) in domain where

$$\begin{cases} f_x(x,y)=0 \\ f_y(x,y)=0 \end{cases}$$ ← system of equations use algebra to solve

(2) If f cts on closed and bdd domain R, then f has global max & min, these occur at one of

(i) critical pts;

(ii) singular pts; or

(iii) boundary of R.

⇒ to find max/min need also to optimize on the boundary.

⇒ if R not closed/not bounded need extra work

Today: Review

① Let $h = ye^{Axy} + B$

<u>Problem</u>: find $h_{xx}, h_{xy}, h_{yx}, h_{yy}$.

$h_x = y \cdot e^{Axy} \cdot Ay = Ay^2 e^{Axy}$ by chain rule

$h_y = e^{Axy} + y \frac{\partial}{\partial y}(e^{Axy}) = e^{Axy} + ye^{Axy}Ax = (1+Axy)e^{Axy}$

pdt rule

chain rule

thus

linearity

chain rule

$h_{xx} = Ay^2 \frac{\partial}{\partial x}(e^{Axy}) = A^2y^3 e^{Axy}$;

$h_{xy} = 2Aye^{Axy} + Ay^2 \frac{\partial}{\partial y}(e^{Axy}) = 2Aye^{Axy} + A^2y^2xe^{Axy}$

pdt rule

$= Ay(2+Axy)e^{Axy}$;

$h_{yx} = Aye^{Axy} + (1+Axy)\frac{\partial}{\partial x}(e^{Axy}) = Aye^{Axy} + (1+Axy)Aye^{Axy}$

chain rule

$= Ay(2+Axy)e^{Axy}$;

$h_{yy} = Axe^{Axy} + (1+Axy)\frac{\partial}{\partial y}(e^{Axy}) = Axe^{Axy} + (1+Axy)Axe^{Axy}$

$= Ax(2+Axy)e^{Axy}$.

Note: $h_{xy} = h_{yx}$.

Notation: $h_{xy} = \frac{\partial^2 h}{\partial y \partial x}$, $h_{xzz} = \frac{\partial^3}{\partial z^2 \partial x} h$, ...

② Agronomy road runs EW under our window.
Say x-axis runs along road, pointing East
y-axis " across " .

$z = z(x,y)$ is height of surface of road

Examples: $\frac{\partial z}{\partial y} = 0 \iff$ street is level

$\frac{\partial z}{\partial x}$ = grade of street = slope

intersection of Agronomy road & Health sciences mall is a local max, has $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0$

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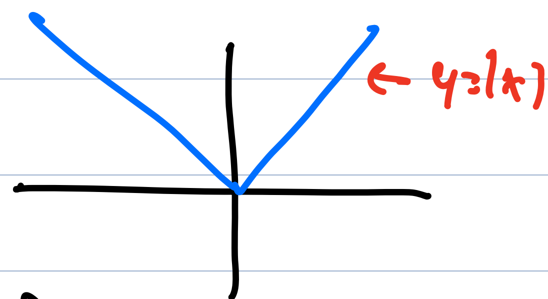
③ $\frac{d}{dx}|x| = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$



AW: $f(x) = \begin{cases} \cos(kx) & |x| < L \\ Ae^{-\beta|x|} & |x| > L \end{cases}$ find f''.

use chain rule

Or: $|x| = \begin{cases} x & x \geq 0 \\ -x & x \leq 0 \end{cases}$

are $\{|x| < L\}$ $\{x < L\}$ same?

$|x| < L$ means: either $x \geq 0$, $x < L$ i.e. $0 \leq x \leq L$
or $x \leq 0$, $-x < L$ $-L \leq x \leq 0$

together: $-L < x < L$

$|x| > L$ means: either $x \geq 0$, then $x > L \iff x > L$
or $x \leq 0$ then $-x > L \iff x < -L$

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⑦ Find critical pts of $f(x) = (7x + 3y + 2y^2)e^{-x-y}$

$$\frac{\partial f}{\partial x} = 7e^{-x-y} - (7x + 3y + 2y^2)e^{-x-y} = (7 - 7x - 3y - 2y^2)e^{-x-y}$$

$$\frac{\partial f}{\partial y} = (3+4y)e^{-x-y} - (7x + 3y + 2y^2)e^{-x-y} = ((3+4y) - (7x+3y+2y^2)) \cdot e^{-x-y}.$$

Need to solve:

$$\begin{cases} (7 - (7x + 3y + 2y^2))e^{-x-y} = 0 \\ ((3+4y) - (7x + 3y + 2y^2))e^{-x-y} = 0 \end{cases}.$$

Know $e^t \neq 0$ for all t, so system is equivalent to:

$$\begin{cases} 7x + 3y + 2y^2 = 7 \\ 7x + 3y + 2y^2 = 3 + 4y. \end{cases}$$

Thus $3 + 4y = 7$ so $\boxed{y = 1}$ and then $7x + 3 + 2 = 7$
so $\boxed{x = \frac{2}{7}}$.

Conclusion: only one critical point, over $(\frac{2}{7}, 1)$.