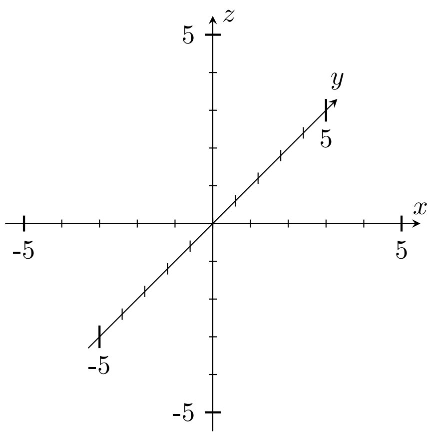
Math 100C – WORKSHEET 10
MULTIVARIABLE CALCULUS

1. Plotting in three dimensions

(1) Plot the points $(2, 1, 3)$, $(-2, 2, 2)$ on the axes provided.



(2) Let $f(x, y) = e^{x^2+y^2}$.

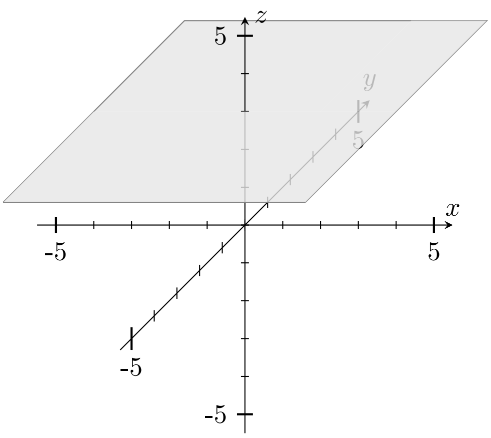
(a) What are $f(0, -1)$? $f(1, 2)$? Plot the point $(0, 1, f(0, 1))$ on the axes provided.

(b) What is the *domain* of f (that is: for what (x, y) values does f make sense?

(c) What is the *range* of f (that is: what values does it take)?

(3) What would the graph of $z = \sqrt{1 - x^2 - y^2}$ look like?

(4) Which plane is this?



(A) $x = 3$
(B) $y = 3$
(C) $z = 3$
(D) none
(E) not sure

Date: 24/11/2022, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

2. Partial derivatives

(5) (a) Let $f(x) = 2x^2 - a^2 - 2$. What is $\frac{df}{dx}$?

(b) Let $f(x) = 2x^2 - y^2 - 2$ where y is a constant. What is $\frac{df}{dx}$?

(c) Let $f(x, y) = 2x^2 - y^2 - 2$. What is the rate of change of f as a function of x *if we keep y constant*?

(d) What is $\frac{\partial f}{\partial y}$?

(6) Find the partial derivatives with respect to both x, y of

(a) $g(x, y) = 3y^2 \sin(x + 3)$

(b) $h(x, y) = ye^{Axy} + B$

(7) One model in labour economics has a production function $Q = \left[\alpha K^\delta + (1-\alpha)E^\delta\right]^{1/\delta}$. Here $\alpha, \delta > 0$ are parameters ($\alpha < 1$), K is the capital and E is the labour.

(a) Find the marginal product of capital: $\frac{\partial Q}{\partial K} =$

(b) Find the marginal product of labour: $\frac{\partial Q}{\partial E} =$

Notations for the partial derivative include $\frac{\partial f}{\partial x}$, $\frac{\partial}{\partial x} f$, $\partial_x f$, $D_x f$,f_x.

(8) We can also compute second derivatives. For example $f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2}{\partial y \partial x} f$. Evaluate:

(a) $h_{xx} = \frac{\partial^2 h}{\partial x^2} =$

(b) $h_{xy} = \frac{\partial^2 h}{\partial y \partial x} =$

(c) $h_{yx} = \frac{\partial^2 h}{\partial x \partial y} =$

(d) $h_{yy} = \frac{\partial^2 h}{\partial y^2} =$

(9) You stand in the middle of a north-south street (say Health Sciences Mall). Let the x axis run along the stree
the south), and let the y axis run across the street. Let $z = z(x, y)$ denote the height of the street surface above

(a) What does $\frac{\partial z}{\partial y} = 0$ say about the street?

(b) What does $\frac{\partial z}{\partial x} = 0.15$ say about the street?

(c) You want to follow the street downhill. Which way should you go?

3. Bonus (<u>**nonexaminable!**</u>): multivariable linear approximation

(10)