Math 100 – WORKSHEET 9
THE CHAIN RULE; INVERSE FUNCTIONS

1. The chain rule

Fact. $(f(g(x)))' = f'(g(x))\,g'(x)$ *or* $\frac{\mathrm{d}}{\mathrm{d}x}(f(g(x))) = \frac{df}{dg}\cdot\frac{dg}{dx}$.

(1) Write the function as a composition and then differentiate.
   (a) e^{3x}

   (b) $\sqrt{2x+1}$

   (c) (Final, 2015) $\sin(x^2)$

   (d) $(7x+\cos x)^n$.

(2) (Final, 2012) Let $f(x) = g(2\sin x)$ where $g'(\sqrt{2}) = \sqrt{2}$. Find $f'\left(\frac{\pi}{4}\right)$.

(3) Differentiate
   (a) $7x + \cos(x^n)$

   (b) $e^{\sqrt{\cos x}}$

   (c) (Final 2012) $e^{(\sin x)^2}$

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(4) Suppose f, g are differentiable functions with $f(g(x)) = x^3$. Suppose that $f'(g(4)) = 5$. Find $g'(4)$.

2. Inverse functions

To find the inverse for $y = f(x)$: (1) "solve for x", get $x = g(y)$ (2) "exchange x, y" to get $g(x)$.

(5) Find the function inverse to $y = x^7 + 3$.

(6) Does $y = x^2$ have an inverse?

(7) Consider the function $y = \sqrt{x-1}$ on $x \geq 1$.
   (a) Find the inverse function, in the form $x = g(y)$.

   (b) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{\mathrm{d}x}{\mathrm{d}y}$ and calculate their product.

To find the derivative of f^{-1}: (1) Convert $y = f^{-1}(x)$ to the form $x = f(y)$ (2) compute $\frac{dx}{dy}$ (3) In $\frac{dy}{dx}$ plug in $y = f^{-1}(x)$ to get expression in terms of x.

(8) Let $f(x) = \log x$. Apply the chain rule to the formula $f(e^y) = y$ to get a formula for $f'(e^y)$, and use that to determine the derivative of the logarithm.

(9) Let $f(x) = x^3 + 5x$. Find $f^{-1}(6)$ and $\left(f^{-1}\right)'(6)$.