Math 100 – SOLUTIONS TO WORKSHEET 8
EXPONENTIAL AND TRIG FUNCTIONS

1. Exponentials

(1) Simplify
  (a) $\left(e^5\right)^3$, $\left(2^{1/3}\right)^{12}$, 7^{3-5}.
  Solution: $\left(e^5\right)^3 = e^{5\cdot 3} = e^{15}$, $\left(2^{1/3}\right)^{12} = 2^{\frac{1}{3}\cdot 12} = 2^4 = 16$, $7^{3-5} = 7^{-2} = \frac{1}{49}$.
  (b) $\log(10e^5)$, $\log(3^7)$.
  Solution: $\log\left(10e^5\right) = \log(10) + 5\log(e) = \log(10) + 5$, $\log(3^7) = 7\log 3$.

(2) Differentiate:
  (a) 10^x
  Solution: This is $(\log 10)\cdot 10^x$.
  (b) $\frac{5\cdot 10^x + x^2}{3^x+1}$
  Solution: By the quotient rule this is

$$\frac{\left(5\log 10\cdot 10^x + 2x\right)\left(3^x+1\right) - \left(5\cdot 10^x + x^2\right)\log 3\cdot 3^x}{\left(3^x+1\right)^2}\,.$$

2. Trigonometric functions

(3) (Special values) What is $\sin\frac{\pi}{3}$? What is $\cos\frac{5\pi}{2}$?
  Solution: $\sin\frac{\pi}{3} = \frac{1}{2}$, $\cos\left(\frac{5\pi}{2}\right) = \cos\left(\frac{\pi}{2}+2\pi\right) = \cos\left(\frac{\pi}{2}\right) = 0$.

(4) Derivatives of trig functions
  (a) Interpret $\lim_{h\to 0}\frac{\sin h}{h}$ as a derivative and find its value.
  Solution: This is $\lim_{h\to 0}\frac{\sin(0+h)-\sin 0}{h} = \frac{d\sin x}{dx}\Big|_{x=0} = \cos 0 = 1$.
  (b) Differentiate $\tan\theta = \frac{\sin\theta}{\cos\theta}$.
  Solution: Applying the quotient rule we get

$$\begin{aligned}\frac{\mathrm{d}\tan\theta}{\mathrm{d}\theta} &= \frac{\cos\theta\cdot\cos\theta - \sin\theta\cdot(-\cos\theta)}{\cos^2\theta}\\ &= \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}\,.\end{aligned}$$

  We also have

$$\frac{\mathrm{d}\tan\theta}{\mathrm{d}\theta} = \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} = 1+\tan^2\theta$$

  which is sometimes useful.

(5) What is the equation of the line tangent the graph $y = T\sin x + \cos x$ at the point where $x = \frac{\pi}{4}$?
  Solution: We have $y(\frac{\pi}{4}) = \frac{T}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{T+1}{\sqrt{2}}$. Also, $\frac{\mathrm{d}y}{\mathrm{d}x} = T\cos x - \sin x$ so $\frac{\mathrm{d}y}{\mathrm{d}x}|_{x=\frac{\pi}{4}} = \frac{T}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{T-1}{\sqrt{2}}$. So the line is

$$y = \frac{T-1}{\sqrt{2}}\left(x-\frac{\pi}{4}\right) + \frac{T+1}{\sqrt{2}}\,.$$

3. Functions in Chains

(6) Write each function as a composition
  (a) e^{3x}
  Solution: This is $f(g(x))$ where $g(x) = 3x$ and $f(y) = e^x$.
  (b) $\sqrt{2x+1}$
  Solution: This is $f(g(x))$ where $g(x) = 2x+1$ and $f(y) = \sqrt{y}$.

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(c) (Final, 2015) $\sin(x^2)$
Solution: This is $f(g(x))$ where $g(x) = x^2$ and $f(y) = \sin y$.

(d) $(7x + \cos x)^n$.
Solution: This is $f(g(x))$ where $g(x) = 7x + \cos x$ and $f(y) = y^n$.