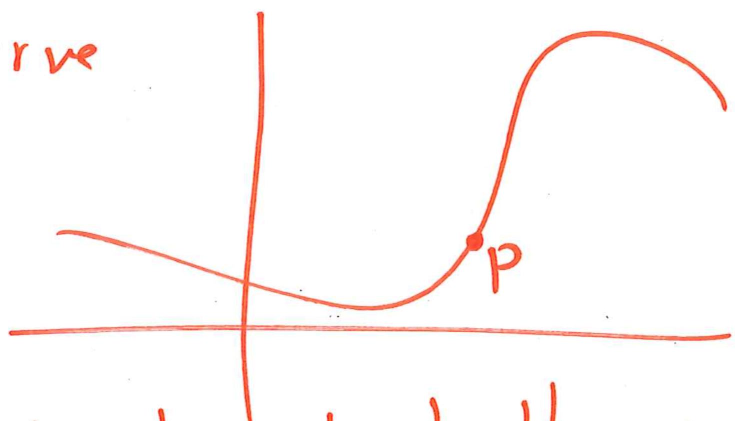
1. Intro; Limits (9/9/2021)

Today's Goals.

(1) Overview of the course
(2) Learning methods
(3) About me
(4) Limits: motivation and first examples

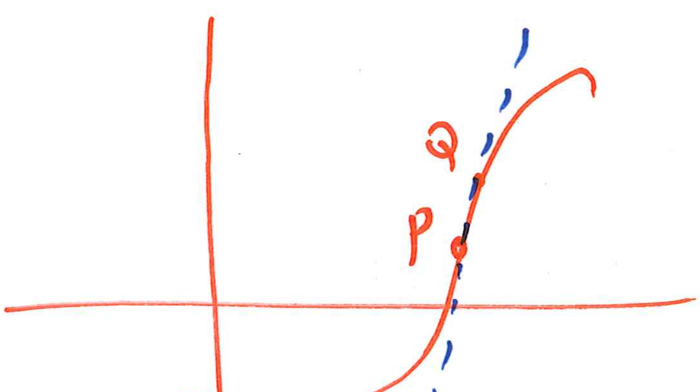
First idea: Limiting processes

Curve



want: line tangent to the curve (at P)

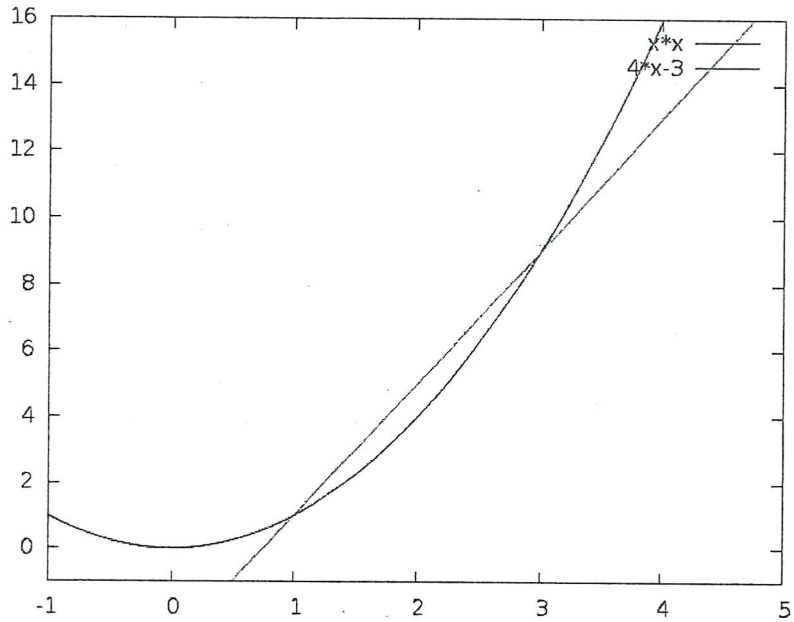
instead draw approximate lines.



warning: conceptual + numerical, not analytic

Math 100 – WORKSHEET 1
LIMITS

1. The slope of a graph



(1) Find the slope of the line through points $P(1,1)$ and $Q(x,x^2)$ where:

(a) $x=3$

$Q(3,9)$

slope : $\frac{\Delta y}{\Delta x} = \frac{9-1}{3-1} = 4.$

(b) $x=1.1$

$\frac{\Delta y}{\Delta x} = \frac{(1.1)^2-1}{1.1-1} = \frac{0.21}{0.1} = 2.1$

(c) $x=1.01$

$\frac{\Delta y}{\Delta x} = \frac{(1.01)^2-1}{1.01-1} = \frac{1.0201-1}{0.01} = 2.01$

(d) $x=1.001$

$\frac{\Delta y}{\Delta x} = \frac{1.002001-1}{1.001-1} = 2.001$

What is the slope of the line tangent to the curve at $P(1,1)$? What is its equation?

slope $=2$, line: $y-1=2(x-1) \iff y=2x-1$

Date: 9/9/2021, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

Redo calculation, point $1+h$

give name to the quantity "how far from 1 are we"

$Q(1+h, 1+2h+h^2)$

slope: $\dfrac{(1+2h+h^2)-1}{(1+h)-1} = \dfrac{2h+h^2}{h} = 2+h \xrightarrow[h\to 0]{} 2$

if $h \neq 0$

the "limit" is the value the expression "wants" to have

(2) Evaluate $f(x) = \frac{x-3}{x^2-x-6}$ at $x = 2.9$, 2.99, 2.999, 3.1,3.01, 3.001. What is $\lim_{x\to 3} f(x)$?

if $x \neq 3$, $\frac{x-3}{x^2-x-6} = \frac{x-3}{(x-3)(x+2)} = \frac{1}{x+2} \underset{x\to 3}{\to} \frac{1}{5}$

(3) Evaluate

(a) $\lim_{x\to 1} \sin(\pi x)$

no surprises: $\sin(\pi x) \underset{x \to 1}{\Rightarrow} \sin(\pi) = 0$

(b) $\lim_{x\to 1} \frac{e^x(x-1)}{x^2+x-2}$.

(c) $\lim_{x\to 0} \frac{\sqrt{1+2x}-\sqrt{1+x}}{3x}$

(4) Either evaluate the limit or explain why it does not exist. Sketching a graph might be helpful.

(a) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 3 & x = 1 \\ 2 - x^2 & x > 1 \end{cases}$.

(b) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 1 & x = 1 \\ 4 - x^2 & x > 1 \end{cases}$.