**Lior Silberman's Math 312: ComPAIR Assignment 4**

- This assignment is due Wednesday, 17/3/2021 at noon (Vancover time)
- Comparisons are due Sunday, 21/3/2021 at 11pm (Vancouver time).

1. (Inverses of multiplicative function)
   (a) Let $f, g$ be non-zero multiplicative functions such that $f(p^k) = g(p^k)$ for all primes $p$ and all $k \geq 1$. Show that $f(n) = g(n)$ for all $n$.
   (b) Let $f$ be a multiplicative function such that $f(1) = 1$, and let $p$ be a prime. Show how to recursively define values $\bar{f}(1), \bar{f}(p), \ldots, \bar{f}(p^r), \ldots$ so that $\left(f * \bar{f}\right)(p^k) = \begin{cases} 1 & k = 0 \\ 0 & k > 0 \end{cases}$.
   RMK A *recursive formula* is the computational analogue of an indutcion: first determine $\bar{f}(1)$. Then for each $r \geq 1$ you need to give a formula for $\bar{f}(p^r)$ in terms of values of $f$ (which is a given in this problem) and the *previous values* $\bar{f}(1), \ldots, \bar{f}(p^{r-1})$.
   (c) In the case where $f = I$ (that is $f(n) = 1$ for all $n$), the inverse is called $\mu$. Apply the recursive formula of part (b) to determine the values $\mu(p^k)$ for all $k$.
   (d) Continuing part (b), define a multiplicative function by $\bar{f}\left(\prod_p p^{e_p}\right) = \prod_p \bar{f}\left(p^{e_p}\right)$. Show it's still true that $\left(f * \bar{f}\right)(p^k) = \delta(p^k)$ for each prime $p$ and prime power $k$.
   (e) In the case $f = I$ show that the reuslting funciton agrees with the Möbius function defined in lecture.
   (f) Continuing part (d), show that $f * \bar{f} = \delta$ (hint: first show that both sides are multiplicative).