## Lior Silberman's Math 312: ComPAIR Assignment 1

- This assignment is due Wednesday, 20/1/2021 at noon (Vancover time)
- Comparisons are due Sunday, $24 / 1 / 2021$ at 11 pm (Vancouver time).

1. You have an infinite supply of $\$ 2$ and $\$ 3$ coins. Use the well-ordering principle to prove that any sum of at least $\$ 2$ can be paid using these coins.
2. Let $f(n)=n^{7}-n$. Show by induction that $f(n)$ is divisible by 7 for all $n$.
3. For any integer $x$ :
(a) Show that $\left(x^{4}+x^{3}+x^{2}+x+1, x-1\right)$ is either 1 or 5 .
(b) Give a concise criterion in terms of $x$ for when the answer is 5 .
(c) Repeat for $\left(x^{5}+x^{4}+x^{3}+x^{2}+x+1, x-1\right)$.
