Math 100 – WORKSHEET 17
THE MEAN VALUE THEOREM

1. More minima and maxima

(1) Show that the function $f(x) = 3x^3 + 2x - 1 + \sin x$ has no local maxima or minima. You may use that $f'(x) = 9x^2 + 2 + \cos x$.

(2) Let $g(x) = xe^{-x^2/8}$ so that $g'(x) = \left(1 - \frac{x^2}{4}\right) e^{-x^2/8}$, find the global minimum and maximum of g on
(a) $[-1, 4]$ (b) $[0, \infty)$

(3) Find the critical numbers and singularities of $h(x) = \begin{cases} x^3 - 6x^2 + 3x & x \leq 3 \\ \sin(2\pi x) - 18 & x \geq 3 \end{cases}$

(4) (Final, 2014) Find a such that $f(x) = \sin(ax) - x^2 + 2x + 3$ has a critical point at $x = 0$.

2. Average slope vs Instantenous slope

(5) Let $f(x) = e^x$ on the interval $[0, 1]$. Find all values of c so that $f'(c) = \frac{f(1)-f(0)}{1-0}$.

(6) Let $f(x) = |x|$ on the interval $[-1, 2]$. Find all values of c so that $f'(c) = \frac{f(2)-f(-1)}{2-(-1)}$

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3. The Mean Value Theorem

Theorem. *Let f be defined and differentiable on $[a, b]$. Then there is c between a, b such that $\frac{f(b)-f(a)}{b-a} = f'(c)$.*
Equivalently, for any x there is c between a, x so that $f(x) = f(a) + f'(c)(x - a)$.

(7) Show that $f(x) = 3x^3 + 2x - 1 + \sin x$ has exactly one real zero. (Hint: let a, b be zeroes of f. The MVT will find c such that $f'(c) =$?)

(8) (Final, 2015)
   (a) Suppose f, f', f'' are all continuous. Suppose f has at least three zeroes. How many zeroes must f', f'' have?
   (b) [Show that $2x^2 - 3 + \sin x + \cos x = 0$ has at least two solutions]
   (c) Show that the equation has at most two solutions.

(9) (Final, 2012) Suppose $f(1) = 3$ and $-3 \leq f'(x) \leq 2$ for $x \in [1, 4]$. What can you say about $f(4)$?

(10) Show that $|\sin a - \sin b| \leq |a - b|$ for all a, b.
(11) Let $x > 0$. Show that $e^x > 1 + x$ and that $\log(1 + x) \leq x$.