Math 100 – SOLUTIONS TO WORKSHEET 16
MINIMA AND MAXIMA

1. Absolute minima and maxima by hand

Theorem. If $f$ is continuous on $[a,b]$ it has an absolute maximum and minimum there.

(1) Find the absolute maximum and minimum values of $f(x) = |x|$ on the interval $[-3, 5]$.

(2) Find the absolute maximum and minimum of $f(x) = \sqrt{x}$ on $[0, 5]$.

2. Derivatives and local extrema

Theorem (Fermat). If, in addition, $f$ is defined and differentiable near $c$ (on both sides!) and has a local extremum at $c$ then $f'(c) = 0$.

Procedure

• Call $c$ a critical point if $f'(c) = 0$, a singular point if $f'(c)$ does not exist.
• To find absolute maximum/minimum of a continuous function $f$ defined on $[a,b]$:
  – Evaluate $f(c)$ at all critical and singular point.
  – Evaluate $f(a), f(b)$.
  – Choose largest, smallest value.

(3) (Final, 2011) Let $f(x) = 6x^{1/5} + x^{6/5}$.
  (a) Find the critical numbers and singularities of $f$.
  (b) Find its absolute maximum and minimum on the interval $[-32, 32]$.

(4) (Final, 2015) Find the critical points of $f(x) = e^{x^{3} - 9x^{2} + 15x - 1}$

*Solution:* By the chain rule

$$f'(x) = f(x) \cdot (3x^{2} - 18x + 15)$$
$$= 3f(x) (x^{2} - 6x + 5)$$
$$= 3f(x) (x - 5) (x - 1).$$

Since $e^y \neq 0$ for all $y$, $f$ is never zero and thus $f'(x) = 0$ iff $(x - 5)(x - 1) = 0$ that is iff $x = 5$ or $x = 1$ and the critical points are 1, 5.

(5) (caution)

(a) Show that $f(x) = (x - 1)^{3} + 7$ attains its absolute minimum at $x = 1$.
(b) Show that $f(x) = (x - 1)^{3} + 7$ has $f'(1) = 0$ but has no local minimum or maximum there.

(6) (Midterm, 2010) Find the maximum value of $x\sqrt{1 - \frac{3}{4}x^{2}}$ on the interval $[0, 1]$.

(7) (Final, 2007) Let $f(x) = x\sqrt{3-x}$.

(a) Find the domain of $f$.
(b) Determine the $x$-coordinates of any local maxima or minima of $f$. 

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