1. **The Linear Approximation**

**Fact:** For $x$ near $a$ we have $f(x) \approx L(x)$ where $L(x) = f(a) + f'(a)(x-a)$

(1) Use a linear approximation to estimate

(a) $\sqrt{1.2}$

(b) (Final, 2015) $\sqrt{8}$

(c) (Final, 2016) $(26)^{1/3}$

(d) log 1.07

2. **Taylor approximation**

(2) Let $f(x) = e^x$

(a) Find $f(0), f'(0), f''(0), \ldots$

(b) Find a polynomial $T_0(x)$ such that $T_0(0) = f(0)$.

(c) Find a polynomial $T_1(x)$ such that $T_1(0) = f(0)$ and $T'_1(0) = f'(0)$.

(d) Find a polynomial $T_2(x)$ such that $T_2(0) = f(0), T'_2(0) = f'(0)$ and $T''_2(0) = f''(0)$.

(e) Find a polynomial $T_3(x)$ such that $T_3^{(k)}(0) = f^{(k)}(0)$ for $0 \leq k \leq 3$.

(3) Do the same with $f(x) = \ln x$ about $x = 1$. 

*Date: 22/10/2019, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.*
Let \( c_k = \frac{f^{(k)}(a)}{k!} \). The \( n \)th order Taylor expansion of \( f(x) \) about \( x = a \) is the polynomial

\[
T_n(x) = c_0 + c_1(x - a) + \cdots + c_n(x - a)^n
\]

(4) Find the 4th order MacLaurin expansion of \( \frac{1}{1-x} \) (Taylor expansion about \( x = 0 \))

(5) Find the \( n \)th order expansion of \( \cos x \).

(6) (Final, 2015) Let \( T_3(x) = 24 + 6(x - 3) + 12(x - 3)^2 + 4(x - 3)^3 \) be the third-degree Taylor polynomial of some function \( f \), expanded about \( a = 3 \). What is \( f''(3) \)?

3. NEW FROM OLD

(7) (Final, 2016) Find the 3rd order Taylor expansion of \( (x + 1) \sin x \) about \( x = 0 \).

(7) Find the 3rd order Taylor expansion of \( \sqrt{x} + 3x \) about \( x = 4 \).

(8) Find the 8th order expansion of \( f(x) = e^{x^2} + \cos(2x) \). What is \( f^{(6)}(0) \)?

(9) Show that \( \log \frac{1+x}{1-x} \approx 2(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots) \). Use this to get a good approximation to \( \log 3 \) via a careful choice of \( x \).