Math 100 – WORKSHEET 6
POLYNOMIALS AND EXPONENTIALS

1. DIRECT PROBLEMS

(1) If \( f, g \) are functions and \( a, b \) are numbers then \( (af + bg)' = af' + bg' \);

(2) \( (fg)' = f'g + fg' \), \( \left( \frac{f}{g} \right)' = \frac{f'g - fg'}{g^2} \);

(3) \( \frac{d}{dx}(x^r) = r x^{r-1} \),

(4) \( \frac{d}{dx}(e^x) = e^x \).

(1) Differentiate

(a) \( f(x) = 6x^π + 2x^e - x^{7/2} \)

(b) (Final, 2016) \( f(x) = x^2 e^x \) (also try \( x^a e^x \))

(c) (Final, 2016) \( f(x) = \frac{x^2 + 3}{2x - 1} \)

(d) \( f(x) = \frac{\sqrt{x(1-3x)}}{x^2 + 1} \)

(e) \( f(x) = \frac{x^2 + xe^x}{\cos x + \sin x} \)
2. Exponentials

(1) Simplify \((e^5)^3, (2^{1/3})^{12}, \pi^3 - 5\).

(2) Differentiate:
   (a) \(10^x\)

   (b) \(\frac{5 \cdot 10^x + x^2}{3x + 1}\)

3. Tangent lines

(1) Suppose that \(f(1) = 1, g(1) = 2, f'(1) = 3, g'(1) = 4\). Find \((fg)'(1)\) and \(\left(\frac{f}{g}\right)'(1)\).

(2) (Final, 2015) Find the equation of the line tangent to the function \(f(x) = \sqrt{x}\) at \((4, 2)\).

(3) Find the lines of slope 3 tangent the curve \(y = x^3 + 4x^2 - 8x + 3\).

(4) Let \(f(x) = \frac{g(x)}{x}\), where \(g(x)\) is differentiable at \(x = 1\). The line \(y = 2x - 1\) is tangent to the graph \(y = f(x)\) at \(x = 1\). Find \(g(1)\) and \(g'(1)\).

(5) (Final 2015) The line \(y = 4x + 2\) is tangent at \(x = 1\) to which function: \(x^3 + 2x^2 + 3x, x^2 + 3x + 2, 2\sqrt{x} + 3 + 2, x^3 + x^2 - x, x^3 + x + 2\), none of the above?