1. **CONTINUITY**

(1) Find $c, d, e$ as appropriate such that each function is continuous on its domain:

\[ f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ c & x = 1 \\ d - x^2 & x > 1 \end{cases} \]  

(Final 2013)

\[ g(x) = \begin{cases} ex^2 + 3 & x \geq 1 \\ 2x^3 - e & x < 1 \end{cases} \]

(2) Where are the following functions continuous?

\[ f(x) = \frac{1}{\sqrt{1 - x^2}}; \quad g(x) = \frac{x^2 + 2x + 1}{2 + \cos x}; \quad h(x) = \frac{2 + \cos x}{x^2 + 2x + 1} \]

(3) (Final 2011) Suppose $f, g$ are continuous such that $g(3) = 2$ and $\lim_{x \to 3} (xf(x) + g(x)) = 1$. Find $f(3)$.

2. **The Intermediate Value Theorem**

**Theorem.** Let $f(x)$ be continuous for $a \leq x \leq b$. Then $f(x)$ takes every value between $f(a), f(b)$.

(1) Show that:

(a) $f(x) = 2x^3 - 5x + 1$ has a zero in $0 \leq x \leq 1$.

(b) $\sin x = x + 1$ has a solution.
(2) (Final 2011) Let \( y = f(x) \) be continuous with domain \([0, 1]\) and range in \([3, 5]\). Show the line \( y = 2x + 3 \) intersects the graph of \( y = f(x) \) at least once.

(3) (Final 2015) Show that the equation \( 2x^2 - 3 + \sin x + \cos x = 0 \) has at least two solutions.

3. DEFINITION OF THE DERIVATIVE

**Definition.** \( f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} \)

(1) Find \( f'(a) \) if
(a) \( f(x) = x^2, \) \( a = 3. \)

(b) \( f(x) = \frac{1}{x}, \) any \( a. \)

(c) \( f(x) = x^3 - 2x, \) any \( a. \) (you may use \((a + h)^3 = a^3 + 3a^2h + 3ah^2 + h^3\)).

(2) Express the limit as a derivative: \( \lim_{h \to 0} \frac{\cos(5+h) - \cos 5}{h}. \)