Math 100 – WORKSHEET 2
LIMIT LAWS

1. Existence of limits and blowup

(1) Let $f(x) = \frac{x-3}{x^2+x-12}$.

    (a) (Final 2014) What is $\lim_{x\to 3} f(x)$?

    (b) What about $\lim_{x\to 2} f(x)$? What about $\lim_{x\to -2^+} f(x)$, $\lim_{x\to 2^-} f(x)$?

    (c) (Final, 2014) Evaluate $\lim_{x\to -3^+} \frac{x+2}{x+3}$.

(2) Evaluate

    (a) $\lim_{x\to 1} \frac{1}{(x-1)^2}$

    (b) $\lim_{x\to \pi^+} \frac{1}{\sin(x)}$, $\lim_{x\to \pi^-} \frac{1}{\sin(x)}$.

(3) Either evaluate the limit or explain why it does not exist. Sketching a graph might be helpful.

    (a) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 1 & x = 1 \\ 2 - x^2 & x > 1 \end{cases}$.

    (b) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 1 & x = 1 \\ 4 - x^2 & x > 1 \end{cases}$.

2. Limit Laws

Fact. *Limits respect arithmetic operations and standard functions (e^x, sin, cos, log, ...)* **as long as everything is well-defined.**

(beware especially of division by zero)

(4) Evaluate using the limit laws:

  (a) $\lim_{x \to 2} \frac{x+1}{4x^2-1} =$

  (b) $\lim_{x \to 1} \frac{e^x (x-1)}{x^2+x-2} =$

(5) Evaluate:

  (a) $\lim_{x \to 0} \frac{\sqrt{4+x}-2}{x}$.

  (b) $\lim_{x \to 0} \frac{\sqrt{1+x}-\sqrt{1+x^2}}{x}$.

  (c) $\lim_{x \to 0} x^2 \sin\left(\frac{\pi}{x}\right)$.

  (d) (Final, 2014) Suppose that $8x \le f(x) \le x^2 + 16$ for all $x \ge 0$. Find $\lim_{x \to 4} f(x)$.