1. The slope of a graph

(1) Find the slope of the line through $P(1,1)$ and $Q(x, x^2)$ where:

(a) $x = 3$
   
   Solution: $Q = (3, 9)$ so slope is $\frac{\Delta y}{\Delta x} = \frac{9 - 1}{3 - 1} = 4$

(b) $x = 1.1$
   
   Solution: $Q = (1.1, 1.21)$ so slope is $\frac{\Delta y}{\Delta x} = \frac{1.21 - 1}{1.1 - 1} = \frac{0.21}{0.1} = 2.1$

(c) $x = 1.01$
   
   Solution: $Q = (1.01, 1.0201)$ so slope is $\frac{\Delta y}{\Delta x} = \frac{1.0201 - 1}{1.01 - 1} = \frac{0.0201}{0.01} = 2.01$

(d) $x = 1.001$
   
   Solution: $Q = (1.001, 1.002001)$ so slope is $\frac{\Delta y}{\Delta x} = \frac{1.002001 - 1}{1.001 - 1} = \frac{0.002001}{0.001} = 2.001$

What is the slope of the tangent line at $P(1,1)$? What is its equation?

Solution: The slope is 2, so the line is $y - 1 = 2(x - 1)$ or $y = 2x - 1$.

2. Limits

(1) Evaluate $f(x) = \frac{x^3}{x^2 - x - 6}$ at $x = 2.9, 2.99, 2.999, 3.1, 3.01, 3.001$. What is $\lim_{x \to 3} f(x)$?

   Solution: For $x \neq 3$ we have $\frac{x^3}{x^2 - x - 6} = \frac{x^3}{(x-3)(x+2)} = \frac{1}{x+2}$ so $\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{1}{x+2} = \frac{1}{5}$

(2) Evaluate

(a) $\lim_{x \to 1} \sin(\pi x)$

   Solution: The function is nice and $\lim_{x \to 1} \sin(\pi x) = \sin(\pi) = 0$. 

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(b) \( \lim_{x \to 1} \frac{e^{x-1}}{x^2 + x - 2} \).

**Solution:** 
\[
\frac{e^{x-1}}{x^2 + x - 2} = \frac{e^{x-1}}{(x-1)(x+2)} \xrightarrow{x \to 1} \frac{e^1}{1+2} = \frac{e}{3}.
\]

(c) \( \lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1+x}}{x} \)

**Solution:** 
We have
\[
\frac{\sqrt{1+2x} - \sqrt{1+x}}{3x} = \frac{\sqrt{1+2x} - \sqrt{1+x}}{3x} \cdot \frac{\sqrt{1+2x} + \sqrt{1+x}}{\sqrt{1+2x} + \sqrt{1+x}}
\]
\[
= \frac{(1+2x) - (1+x)}{3x(\sqrt{1+2x} + \sqrt{1+x})}
\]
\[
= \frac{x}{3x(\sqrt{1+2x} + \sqrt{1+x})}
\]
\[
= \frac{1}{3(\sqrt{1+2x} + \sqrt{1+x})} \xrightarrow{x \to 0} \frac{1}{3(\sqrt{1} + \sqrt{1})} = \frac{1}{6}.
\]

(3) Either evaluate the limit or explain why it does not exist. Sketching a graph might be helpful.

(a) \( \lim_{x \to 1} f(x) \) where \( f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ 1 & x = 1 \\ 2 - x^2 & x > 1 \end{cases} \).

**Solution:** From the left \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \sqrt{x} = \sqrt{1} = 1 \). From the right \( \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2 - x^2 = 2 - 1 = 1 \) so the limit exists and equals 1.

(b) \( \lim_{x \to 1} f(x) \) where \( f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ 1 & x = 1 \\ 4 - x^2 & x > 1 \end{cases} \).

**Solution:** From the left \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \sqrt{x} = \sqrt{1} = 1 \). From the right \( \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 4 - x^2 = 4 - 1 = 3 \) so the does not exist.