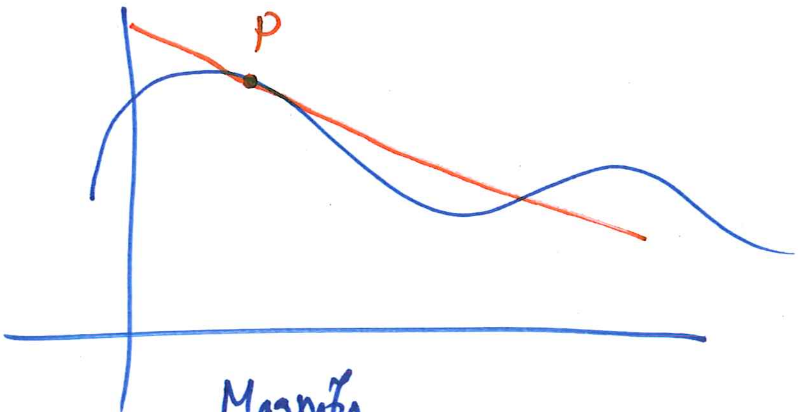
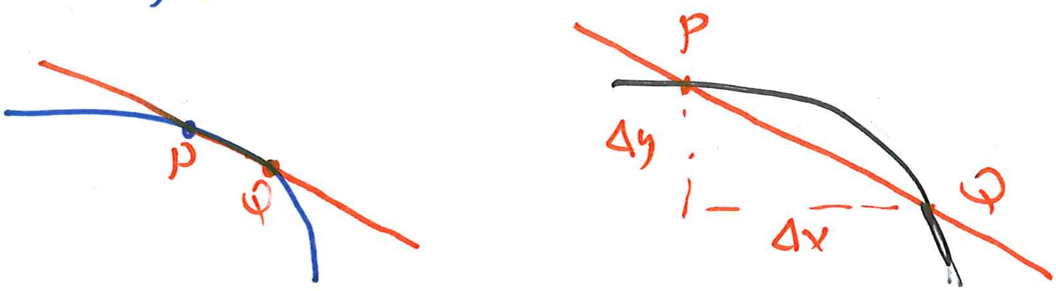
Math 100, lecture 1: Limits! 5/9/2019

Motivation: Look at the line tangent to a curve.



Magnify



Idea: ~~Make~~ Can't draw tangent line directly.
We can draw secant lines (connect P to Q)
then shift Q toward P.

Worksheet 1

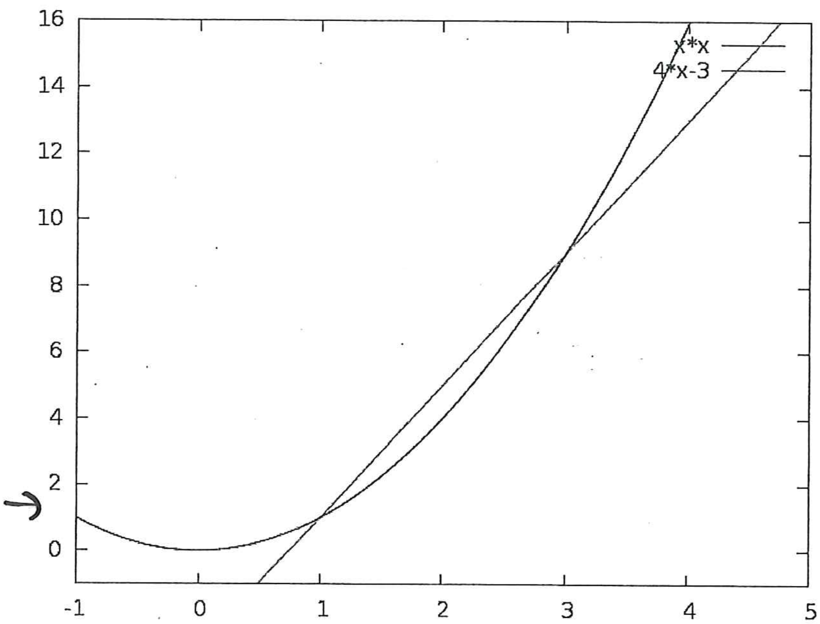
Idea: used limiting process to find the slope

Limit: We have a function $g(x)$, defined near $x = x_0$
(here $g(x)$ = slope of secant line, $x_0 = 1$), take x close to x_0
~~to~~ if this makes $g(x)$ close to value a ($a = 2$ here)
say "g tends to a as x tends to x_0".

Math 100 – WORKSHEET 1
TANGENT AND VELOCITY PROBLEMS; LIMITS

1. The slope of a graph

$y = x^2$ →



(1) Find the slope of the line through $P(1,1)$ and $Q(x, x^2)$ where:

(a) $x = 3$ $\quad Q = (3,9)$ slope $= \frac{\Delta y}{\Delta x} = \frac{9-1}{3-1} = \frac{8}{2} = 4$

(b) $x = 1.1$ $\quad \frac{(1.1)^2 - 1}{1.1 - 1} = \frac{1.21 - 1}{1.1 - 1} = \frac{0.21}{0.1} = 2.1$

(c) $x = 1.01$ $\quad \frac{(1.01)^2 - 1}{1.01 - 1} = \frac{1.0201 - 1}{1.01 - 1} = \frac{0.0201}{0.01} = 2.01$

(d) $x = 1.001$

$= 2.001$

What is the slope of the tangent line at $P(1,1)$? What is its equation?

slope $= 2$ line: $y - 1 = 2(x-1) \iff y = 2x - 1$

Date: 5/9/2019, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

2. Limits

(1) Evaluate $f(x) = \frac{x-3}{x^2-x-6}$ at $x = 2.9,\ 2.99,\ 2.999,\ 3.1, 3.01,\ 3.001$. What is $\lim_{x\to 3} f(x)$?

$f(2.9) \approx 0.204$ $\quad f(2.99) \approx 0.2004$, $\quad f(2.999) \approx 0.20004$

$f(3.1) \approx 0.196$, $\quad f(3.01) \approx 0.1996$, $\quad f(3.001) \approx 0.19996$

Guess: $\lim_{x\to 3} \frac{x-3}{x^2-x-6} = 0.2 = \frac{1}{5}$

Note: $\frac{x-3}{x^2-x-6} = \frac{x-3}{(x-3)(x+2)} = \frac{1}{x+2}$

so $\lim_{x\to 3} \frac{x-3}{x^2-x-6} = \lim_{x\to 3} \frac{1}{x+2} = \frac{1}{3+2} = \frac{1}{5}$

(2) Evaluate

(a) $\lim_{x\to 1} \sin(\pi x)$

(b) $\lim_{x\to 1} \frac{e^x(x-1)}{x^2+x-2}$.

(c) $\lim_{x\to 0} \frac{\sqrt{1+2x}-\sqrt{1+x}}{3x}$