1. Find $\frac{dy}{dx}$ by implicit differentiation if 

$$e^{x/y} = x - y.$$ 

**Solution:** We differentiate both sides, and find:

$$1 - \frac{dy}{dx} = \frac{d}{dx} \left( e^{x/y} \right) = e^{x/y} \frac{d}{dx} \left( \frac{x}{y} \right) = e^{x/y} \left( \frac{1}{y} \right) \frac{d}{dx} \left( 1 - \frac{x}{y^2} \right).$$

Rearranging this reads:

$$\left[ \frac{x}{y^2} e^{x/y} \right] \frac{dy}{dx} = 1 - \frac{1}{y} e^{x/y} - 1$$

that is

$$\frac{dy}{dx} = \frac{\frac{1}{y} e^{x/y} - 1}{\frac{x}{y^2} e^{x/y} - 1} = \frac{ye^{x/y} - y^2}{xe^{x/y} - y^2}.$$ 

2. Differentiate 

$$y = \sqrt{\arctan x}.$$ 

**Solution:** We can write this as $\arctan x = y^2$ and hence $\tan y^2 = x$. We now differentiate, using that $\frac{d(\tan y)}{dy} = 1 + \tan^2 y$ to get

$$2y(1 + \tan^2 y^2) \frac{dy}{dx} = \frac{dx}{dx} = 1$$

so

$$\frac{dy}{dx} = \frac{1}{2y(1 + \tan^2 y^2)} = \frac{1}{2\sqrt{\arctan x}(1 + x^2)}.$$ 

3. Find $y'$ at the point where $x = 0$ if 

$$xy + e^y = e.$$ 

**Solution:** Setting $x = 0$ we find $e^{y(0)} = e = e^1$ so $y(0) = 1$ and the point on the curve is $(0, 1)$. We now differentiate both sides, using the product rule and the chain rule to get:

$$y + x \frac{dy}{dx} + e^y \frac{dy}{dx} = 0$$

so

$$\frac{dy}{dx} = -\frac{y}{x + e^y}.$$ 

When $x = 0$ and $y = 1$ this reads

$$y'(0) = -\frac{1}{e}.$$