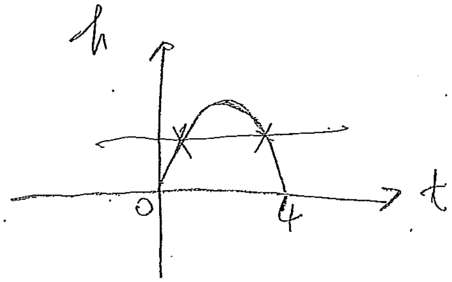
Solutions to Math 184 Mid-term 1. $\boxed{\text{(Version I)}}$

(Oct. 16, 2018)

1. (a) [3.]



$\boxed{\text{Answer: NO}}$

(b) [3]. $\lim_{x \to a^+} f(x) = \frac{M}{3}$.

$\boxed{\text{Answer: } L = \frac{M}{3}}$

(c) [3] $A = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} 3 = 3$.

$B = \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+5) = 3+5 = 8$

$C = \lim_{x \to 0} f(x) = \lim_{x \to 0} 3 = 3$.

$\boxed{\text{Answer: } A=3,\ B=8,\ C=3.}$

(d) [3] $g' = (x^2)' f + x^2 f' = 2x f(x) + x^2 f'(x)$

$g'(2) = 2 \cdot 2 f(2) + 2 \cdot 2 f'(2) = 4(-1) + 4(3) = 8$

$\boxed{\text{Answer: } 8}$

(e) [3] $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} = \lim_{x \to 25} \frac{(\sqrt{x} - 5)(\sqrt{x}+5)}{(x-25)(\sqrt{x}+5)} = \lim_{x \to 25} \frac{x-25}{(x-25)(\sqrt{x}+5)} = \lim_{x \to 25} \frac{1}{\sqrt{x}+5}$

$= \frac{1}{\sqrt{25}+5} = \frac{1}{10}$.

$\boxed{\text{Answer: } \frac{1}{10}}$

or $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} = \lim_{x \to 25} \frac{\sqrt{x} - 5}{(\sqrt{x}+5)(\sqrt{x}-5)} = \lim_{x \to 25} \frac{1}{\sqrt{x}+5} = \frac{1}{10}$.

2. [5] $$f'(-1) = \lim_{x\to -1} \frac{f(x)-f(-1)}{x-(-1)} = \lim_{x\to -1} \frac{\frac{3}{5x+2} - \frac{3}{5(-1)+2}}{x+1}$$

$$= \lim_{x\to -1} \frac{\frac{3}{5x+2}+1}{x+1} = \lim_{x\to -1} \frac{\left(\frac{3}{5x+2}+1\right)(5x+2)}{(x+1)(5x+2)} = \lim \frac{3+(5x+2)}{(x+1)(5x+2)}$$

$$= \lim_{x\to -1} \frac{5x+5}{(x+1)(5x+2)} = \lim_{x\to -1} \frac{5(x+1)}{(x+1)(5x+2)} = \lim_{x\to -1} \frac{5}{5x+2} = \frac{5}{5(-1)+2} = -\frac{5}{3}.$$ //

3. (a) [5] Let $f(x) = \sin\left(\frac{\pi}{4}x\right) - x^2 + \frac{1}{2}$.

$$f(0) = \sin 0 - 0 - \tfrac{1}{2} > 0,$$

$$f(2) = \sin\left(\tfrac{\pi}{2}\right) - 2^2 + \tfrac{1}{2} = 1 - 4 + \tfrac{1}{2} < 0$$

Since $f(x)$ is continuous on $[0, 2]$, (By IVT), there is a solution to the equation $\sin\left(\frac{\pi}{4}x\right) = x^2 - \frac{1}{2}$ in the interval $(0, 2)$ (or $[0, 2]$). //

(b) [3] $$\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{x^2+3x+2}{x+1} = \lim_{x\to -1} \frac{(x+1)(x+2)}{x+1} = \lim_{x\to -1}(x+2) = 1$$

Hence, $f(x)$ is continuous at $x=-1$ if and only if $1 = \lim_{x\to -1} f(x) = f(-1) = a$. Hence, we need to choose a such that $a = 1$. //

5-1

4. (a).[3] $h'(x) = \left(\frac{f(x)}{x-3}\right)' = \frac{f'(x)(x-3) - f(x)(x-3)'}{(x-3)^2}$

$$= \frac{f'(x)(x-3) - f(x)}{(x-3)^2}$$

$$h'(2) = \frac{f'(2)(2-3) - f(2)}{(2-3)^2} = 3(-1) - 2 = -5.$$

(b) [3] $p(x) = (x-4)(x-c)$ for some constant c

$\left(\text{because } p(4) = \lim_{x\to 4} p(x) = \lim_{x\to 4} \frac{p(x)}{x-4}(x-4) = \lim_{x\to 4} \frac{p(x)}{x-4} \lim_{x\to 4}(x-4) = 3\cdot 0 = 0 \right)$

We get $3 = \lim_{x\to 4} \frac{p(x)}{x-4} = \lim_{x\to 4} \frac{(x-4)(x-c)}{x-4}$

$$= \lim_{x\to 4}(x-c) = 4-c \Rightarrow c = 1.$$

Hence, $x^2 + ax + b = (x-4)(x-c) = (x-4)(x-1) = x^2 - 5x + 4$

$\Rightarrow a = -5$ and $b = 4$

5. (a) [3] Let $p = f(q) = mq + b$ be the demand function, where m and b are constants. Since $(40, 14)$ and $(25, 17)$ are on the graph of $p = f(q)$, we get

$$m = \frac{17-14}{25-40} = -\frac{1}{5}$$

Hence, we get

$$p - 14 = -\frac{1}{5}(q-40) \quad \text{or} \quad p = -\frac{1}{5}q + 22.$$

(b) [3]

$$C(q) = 500 + 8q$$
$$R(q) = pq = \left(-\frac{1}{5}q + 22\right)q = -\frac{1}{5}q^2 + 22q$$
$$P(q) = R(q) - C(q) = -\frac{1}{5}q^2 + 14q - 500.$$

(c) [2] The owner gets the maximal profit when

$$q = -\frac{14}{2(-\frac{1}{5})} = 35.$$

Hence, the price the owner should set is

$$p(35) = -\frac{1}{5}(35) + 22 = -7 + 22 = \$15.$$

(d) [3] If $p = 10$, we have $10 = -\frac{1}{5}q + 22 \Rightarrow q = 60$.

Since $\frac{dR}{dq} = -\frac{2}{5}q + 22$, we get $\left.\frac{dR}{dq}\right|_{q=60} = -\frac{2}{5}(60) + 22 < 0$

Hence, we know that $p \downarrow \Rightarrow q \uparrow \Rightarrow R \downarrow$, i.e. if the price is decreased by a small amount, the weekly revenue decreases.

$\frac{D}{(-4}$

6. (a)[3] $$L = \lim_{h\to 0}\left(\frac{(1+h)^{2018}-1}{h} + \frac{2(1+h)^{2017}-2}{h}\right)$$

Hence, we have two choices (at least):

Choice I: $f(x) = x^{2018}$, $g(x) = 2x^{2017}$, $c = 1$.

(or $f(x) = x^{2018} + b$, $g(x) = 2x^{2017} + b$, where b is a constant)

Choice II: $f(x) = 2x^{2017}$, $g(x) = x^{2018}$, $c = 1$.

(b)[2] If you use the choice I, then

$$f'(x) = 2018x^{2017}, \quad \frac{1}{2}g'(x) = \frac{1}{2}\cdot 2(2017)x^{2016}$$

$$\Rightarrow f'(-1) + \frac{1}{2}g'(-1) = 2018(-1)^{2017} + 2017(-1)^{2016} = -1$$

If you use the choice II, then

$$f'(x) = 2(2017)x^{2016}, \quad g'(x) = 2018x^{2017}$$

$$\Rightarrow f'(-1) + \frac{1}{2}g'(-1) = 2(2017)(-1)^{2016} + \frac{1}{2}(2018)(-1)^{2017}$$

$$= 4034 - 1009 = 3025$$