1. First, observe that if $G$ is a graph that has no vertices of degree two, then $G$ is a maximal reduction of itself.  
Next, note that if $G_1$ and $G_2$ are graphs, neither of which contain any vertices of degree two, then $G_1$ and $G_2$ are homeomorphic if and only if they are isomorphic. Indeed, if $G_1$ and $G_2$ are isomorphic then they are clearly homeomorphic. In the order direction, if $G_1$ and $G_2$ are homeomorphic, then $G_2$ can be obtained from $G_1$ by deleting vertices of degree 2 (and “joining” the corresponding edges), or inserting vertices in the “middle” of edges. These operations either remove a vertex of degree 2 or add a vertex of degree 2. Since neither $G_1$ nor $G_2$ contain vertices of degree 2, we conclude that $G_1$ and $G_2$ must be isomorphic. 
Finally, let $G_1$ and $G_2$ be graphs, and let $G'_1$ and $G'_2$ be their maximal reductions, respectively. Since $G_1$ is homeomorphic to $G'_1$, $G_2$ is homeomorphic to $G'_2$, and the property of being homeomorphic is transitive, $G_1$ is homeomorphic to $G_2$, $G'_1$ and $G'_2$ are isomorphic (and hence homeomorphic). On the other hand, if $G_1$ and $G_2$ are homeomorphic then $G'_1$ and $G'_2$ must be homeomorphic, which implies that they are isomorphic.

2. The LHS one is, and the RHS one isn’t.

3. In order to get from graph $G_1$ to $G_2$ we repeatedly either add in a vertex to an edge increasing both the number of vertices and number of edges by 1, or delete a vertex of degree 2 decreasing both the number of vertices and number of edges by 1. Hence, repeatedly, the (number of vertices)-(number of edges) stays constant, or $m_1 - n_1 = m_2 - n_2$.

4. Let us do a proof by contradiction and assume that such a polyhedral graph $G$ exists. Let $v$ denote the number of vertices in $G$, then since $G$ is polyhedral it satisfies Euler’s theorem and hence $v - 24 + 8 = 2$ and so $v = 18$.

Since every vertex has degree at least 3 we know from counting degrees that $3v \leq 2e$ since every edge has 2 ends. Hence since $v = 18$ and $e = 24$ substituting this in we have that $54 = 3.18 \leq 2.24 = 48$, a contradiction. Therefore no such polyhedral graph exists.
5. If $G$ is regular of degree $k > 0$ then every vertex has degree $k$ so every edge meets $k - 1$ other edges at a vertex it is incident to. Every edge is incident to 2 vertices, that is has 2 ends, and hence every edge meets $2k - 2$ other edges in total. Thus in $L(G)$ every vertex has degree $2k - 2$ by definition, so $L(G)$ is regular of degree $2k - 2$.

6. In $G$ let $e$ be an edge incident to $v_i$ of degree $d_i$. Then in $L(G)$ the vertex corresponding to $e$, $v_e$, will be adjacent to $(d_i - 1)$ other vertices by definition of $L(G)$. Since there are $d_i$ possibilities for $e$, each vertex $v_i$ generates $d_i(d_i - 1)$ edge ends in $L(G)$ and hence there are $\sum_{i=1}^{n} d_i(d_i - 1)$ in $L(G)$ overall. Since every edge has 2 ends, the total number of edges in $L(G)$ is

$$\sum_{i=1}^{n} \frac{d_i(d_i - 1)}{2}.$$ 

7. We have that the bipartite graph $K_{1,n}$ for $n \geq 3$ satisfies $L(K_{1,n}) = K_n$. This is because in $K_{1,n}$ every edge meets the $n - 1$ others, and hence in $L(K_{1,n})$ every vertex is adjacent to the $n - 1$ others. This is $K_n$ by definition.