

Math 341 Homework 3

- Due Thursday, March 1 at start of class.
- If your homework is longer than one page, **staple** the pages together, and put your name on each sheet of paper.
- You are allowed (and encouraged) to use results proved in class on your homework.
- **Collaboration Policy:** You are welcome (and encouraged) to work on the homework in groups. However, each student must write up the homework on their own, and must use their own wording (i.e. don't juse copy the solutions from your friend). If you do collaborate with others, please list the name of your collaborators at the top of the homework.
- You are encouraged (though not required) to type up your solutions. If you choose to do this, I strongly recommend that you use the typesetting software LaTeX. LaTeX is used by the entire mathematics community, and if you intend to go into math, youll need to learn it sooner or later. "The Not So Short Introduction to LaTeX" is a good place to start. This guide can be found at <http://tug.ctan.org/info/lshort/english/lshort.pdf> . You can also download the .tex source file for this homework and take a look at that.
- Each homework problem should be correct as stated. Occasionally, however, I might screw something up and give you an impossible homework problem. If you believe a problem is incorrect, please email me. If you are right, the first person to point out an error will get +1 on that homework, and I will post an updated version.

Fibonacci numbers

1. Prove that if n is divisible by three, then F_n is even.
2. Prove that if $n \geq 6$ and n is even, then F_n is composite (i.e. it is not prime). Hint: try expanding out the formula $F_n = F_{n-1} + F_{n-2}$ multiple times.

Generating functions

3. Expand $\frac{2t}{1-8t+15t^2}$ as a power series (i.e. write it in the form $\sum_{n \geq 0} a_n t^n$, and compute the numbers a_n).
4. Consider the sequence a_n defined by $a_0 = 0$ and $a_{n+1} = 3a_n + 2$ for $n \geq 0$. Using the method of generating functions, write down a formula for a_n .
5. Consider the sequence a_n defined by $a_0 = 0$, $a_1 = 1$, and $a_{n+2} = 2a_{n+1} - a_n$ for $n \geq 0$. Using the method of generating functions, write down a formula for a_n .

Permutations

6. Let $\pi \in S_n$. Prove the formula $\text{sgn}(\pi) = (-1)^{n-C(\pi)-F(\pi)}$ from lecture.

7. a. Let $\pi \in S_n$ be of the form $\pi = (a_1 a_2 \dots a_k)$. Prove that π is even if and only if k is odd. For part b, we will call k the “length” of the cycle.

b. Let $\pi \in S_n$, and consider a representation of π as a product of disjoint cycles. Prove that π is even if and only if there are an even number of even length cycles (and any number of odd length cycles).