Math 120 Midterm 2 Practice 2

This midterm is 50 minutes, closed book, no calculators, phones, etc. Solve each problem using the paper provided, and put your full name at the top of each sheet of paper. No clarification will be given for any problems; if you believe a problem is ambiguous, interpret it as best you can and write down any assumptions you feel are necessary.

Name: ______________________________

Student #: ______________________________

Problem	Score
1:	/10
2:	/10
3:	/10
4:	/10
Total:	/40

Name: ____________________

1.

a) Define

$$f(x) = \begin{cases} x^2, & x \neq 0, \\ 1, & x = 0 \end{cases}$$

What type of discontinuity does f have at 0? You do not need to prove that your answer is correct.

b) Define

$$f(x) = \begin{cases} x^2, & x \geq 0, \\ 1 - x^2, & x < 0 \end{cases}$$

What type of discontinuity does f have at 0? You do not need to prove that your answer is correct.

c) Define $f(x) = 1/x^2$. What type of discontinuity does f have at 0? You do not need to prove that your answer is correct.

d) Define $f(x) = x^2$. What type of discontinuity does f have at 0? You do not need to prove that your answer is correct.

Name: ____________________________

2. (10 points) Recall that a function f is called injective (or one-to-one) if for all $x, y \in D(f)$ with $x \neq y$, we have $f(x) \neq f(y)$.

a) Let f be a function that is differentiable at every point $x \in \mathbb{R}$, and suppose that $f'(x) > 0$ for all $x \in \mathbb{R}$. Must it be true that f is injective? If yes, then prove it. If not, then give an example of a function f for which the statement is not true, and show that your example is correct.

b) Let f be a function that is differentiable at every point $x \in \mathbb{R}$, and suppose that $f'(x) \geq 0$ for all $x \in \mathbb{R}$. Must it be true that f is injective? If yes, then prove it. If not, then give an example of a function f for which the statement is not true, and show that your example is correct.

Name: ____________________

3. (10 points). Use Newton's method to approximate the positive square root of seven, i.e. $\sqrt{7}$.

To do this, write down a function with integer coefficients for which $\sqrt{7}$ is a root, choose an appropriate starting guess x_0, and then iterate twice to determine x_1 and x_2. You do not need to simplify your fractions. You will be evaluated based on your choice of function, your choice of starting point, and the correct application of Newton's method.

Name: ____________________

(scratch space for problem 3)

Name: ____________________

4. (10 points) Let f be a function whose domain is $\mathbb{R}$. Suppose that for all $x, y \in \mathbb{R}$, we have $|f(x) - f(y)| \leq |x - y|^2$. Prove that f is constant, i.e. there is a number $C \in \mathbb{R}$ so that $f(x) = C$ for all $x \in \mathbb{R}$.

Name: ____________________________

(scratch space for problem 4)