Math 120 Midterm 2 Practice 1

This midterm is 50 minutes, closed book, no calculators, phones, etc. Solve each problem using the paper provided, and put your full name at the top of each sheet of paper. No clarification will be given for any problems; if you believe a problem is ambiguous, interpret it as best you can and write down any assumptions you feel are necessary.

Name: ______________________________

Student #: ______________________________

Problem	Score
1:	/10
2:	/10
3:	/10
4:	/10
Total:	/40

Name: ________________________

Student #: ________________________

1. Let (a, b) be an interval and let $c \in (a, b)$. Let f be a function that is continuous on $(a, b)\backslash\{c\}$, and suppose f has an infinite discontinuity at c. Must it be true that $\lim_{x\to c} |f(x)| = \infty$? If the answer is yes, prove it. If no, give a counter-example and prove that your counter-example is correct.

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Student #: ____________________

2. Let f be a function that is continuous on the interval $[a, b]$ and differentiable on the interval (a, b). Suppose that there exists $M > 0$ so that for each $x \in (a, b)$, we have $|f'(x)| \leq M$. Prove that $|f(b) - f(a)| \leq M|b - a|$.

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3. The number $\frac{1+\sqrt{5}}{2}$ is called the golden ratio.

a) What are the two integers closest to the golden ratio? You do not have to prove that your answer is correct.

b) Use Newton's method to approximate the golden ratio. Start with one of the two integers you gave above, and iterate Newton's method two times (i.e. you should have your starting guess and then two refinements of this guess). You do not need to simplify fractions.

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(scratch space for problem 3)

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4. Define

$$f(x) = \begin{cases} x^2, & x \in \mathbb{Q}, \\ -x^2, & x \in \mathbb{R}\backslash\mathbb{Q} \end{cases}$$

a) Prove that f is differentiable at 0.

b) Prove that for each $c \neq 0$, f is *not* differentiable at c.

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(scratch space for problem 4)