

Math 120 Homework 4

- Due Friday October 12th at 10 am. Homework should be submitted using Canvas.
- **Collaboration Policy:** You are welcome (and encouraged) to work on the homework in groups. However, each student must write up the homework on their own, and must use their own wording (i.e. don't just copy the solutions from your friend). If you do collaborate with others, please list the name of your collaborators at the top of the homework.
- You are encouraged (though not required) to type up your solutions. If you choose to do this, I strongly recommend that you use the typesetting software LaTeX. LaTeX is used by the entire mathematics community, and if you intend to go into math, you'll need to learn it sooner or later. "The Not So Short Introduction to LaTeX" is a good place to start. This guide can be found at <http://tug.ctan.org/info/lshort/english/lshort.pdf>. You can also download the .tex source file for this homework and take a look at that.
- Each homework problem should be correct as stated. Occasionally, however, I might screw something up and give you an impossible homework problem. If you believe a problem is incorrect, please email me. If you are right, the first person to point out an error will get +1 on that homework, and I will post an updated version.

1. Let $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ and let $Q(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_0$ be non-zero polynomials (i.e. either P nor Q is identically 0). Suppose that $a_n > 0$ and $b_m > 0$, and that $n > m$. Prove that

$$\lim_{x \rightarrow \infty} (P/Q)(x) = \infty.$$

Note: This result was stated in class, but we didn't prove it. Your job now is to prove it.

2. Give an example of functions f and g so that $\lim_{x \rightarrow 1} g(x) = 1$ and $\lim_{x \rightarrow 1} f(x) = 1$, but $\lim_{x \rightarrow 1} f \circ g(x)$ is not equal to 1.

3. Let f and g be functions whose domain is \mathbb{R} . Suppose that $f(1) = 1$, $\lim_{x \rightarrow 1} g(x) = 1$, and $\lim_{x \rightarrow 1} f(x) = 1$. Prove that

$$\lim_{x \rightarrow 1} f \circ g(x) = 1.$$

4. This problem shows a relationship between one-sided limits and limits at infinity. Let $g(x) = 1/x$. Let $f(x)$ be a function and suppose that $(0, \infty) \subset D(f)$.

a. Suppose that $\lim_{x \rightarrow 0^+} f(x) = L$. Prove that $\lim_{x \rightarrow \infty} f \circ g(x) = L$ (don't forget to establish *both* requirements for the limit to exist).

b. Suppose that $\lim_{x \rightarrow \infty} f \circ g(x) = L$. Prove that $\lim_{x \rightarrow 0^+} f(x) = L$.

5. (This problem requires the definition of continuity, which will be discussed in lecture on Monday October 7). Let f and g be functions, and suppose f and g are continuous on the open interval $(-\infty, \infty) = \mathbb{R}$ (in particular, this means $D(f) = \mathbb{R}$, $D(g) = \mathbb{R}$). Suppose that for every rational number $x \in \mathbb{Q}$, we have $f(x) = g(x)$. Prove that $f(x) = g(x)$ for *every* number $x \in \mathbb{R}$.