Math 120 Homework 10

- Not to be handed in. However, you are strongly encouraged to complete this homework, since this material will be part of the final exam.

1. Graph the function $f(x) = \arctan(\tan(x))$ for all x in $D(f) \cap [-4\pi, 4\pi]$. You do not need to prove that your graph is correct, but draw it carefully—mark your x and y axes, scale things properly, and be sure to get the domain correct.

2. Prove that
$$\sinh(x + y) = \sinh(x) \cosh(y) + \cosh(x) \sinh(y),$$
and
$$\cosh(x + y) = \cosh(x) \cosh(y) + \sinh(x) \sinh(y).$$

3. At which points on the ellipse $x^2 + 3y^2 = 1$ is the tangent line parallel to the line $y = x$? Prove that your answer is correct.

4. Compute the slope of the tangent line of the curve $y^5 + 2xy^3 + 3x^2y + 10x = 16$ at the point $(1, 1)$.

5. Use L'Hopital's rule to prove the following

(a) Prove that if f and g are polynomials with $f(x) = a_n x^n + \ldots + a_0$ and $g(x) = b_n x^n + \ldots + b_0$, with $a_n \neq 0$, $b_n \neq 0$, then
$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = a_n/b_n.$$

(b) Prove that if f and g are polynomials with $\deg(g) > \deg(f)$, then

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0.$$