Math 120 Homework 6

- Due Friday October 27 at start of class.
- If your homework is longer than one page, **staple** the pages together, and put your name on each sheet of paper. **Homework lacking a staple will lose 2 points**.
- Each homework problem should be correct as stated. Occasionally, however, I might screw something up and give you an impossible homework problem. If you believe a problem is incorrect, please email me. If you are right, the first person to point out an error will get +1 on that homework, and I will post an updated version.

1. We would like to use Newton's method to compute $\sqrt[3]{3}$. Write down a function $f(x)$ so that $f(\sqrt[3]{3}) = 0$, and a starting guess x_1 that is a rational number so that Newton's method is guaranteed to work with this choice of f and x_1. Prove that your answer is correct.

2. Let f be a function that is continuous on $\mathbb{R}$ and 2π periodic: This means that for all $x \in \mathbb{R}$, $f(x) = f(x + 2\pi)$. Let $g(x) = f(x + \pi) - f(x)$.

a. Show that there is $a \in \mathbb{R}$ such that $g(a) = 0$.
b. Deduce that on the equator, there are always two points diametrically opposed with the same temperature.

3. Let f and g be functions, and let $a, b \in \mathbb{R}$. Suppose that g is continuous at a and $g(a) = b$. Suppose furthermore that f is continuous at b. Prove that $f \circ g$ is continuous at a.

Remark. This problem is a generalization of HW 4 #1, so taking a look at that could be useful. Since some students were having trouble with this, I decided to ask it again.

4 a. Prove that for each even natural number $n \in \mathbb{N}$, $n! \geq (n/2)^{n/2}$.
b. Prove that for each $x \in \mathbb{R}$, and each $\epsilon > 0$, there exists a number $n \in \mathbb{N}$ so that $\left|\frac{x^n}{n!}\right| < \epsilon$.
c. Prove that for each $x > 0$ and each $\epsilon > 0$, there exists a number $n \in \mathbb{N}$ so that

$$\left| \cos(y) - 1 - \sum_{k=1}^{n} \frac{(-1)^k}{(2k)!} y^{2k} \right| < \epsilon.$$

for all $y \in [-x, x]$.