Math 120 Homework 4

- Due Friday October 13 at start of class.
- If your homework is longer than one page, **staple** the pages together, and put your name on each sheet of paper. **Homework lacking a staple will lose 2 points**.
- Each homework problem should be correct as stated. Occasionally, however, I might screw something up and give you an impossible homework problem. If you believe a problem is incorrect, please email me. If you are right, the first person to point out an error will get +1 on that homework, and I will post an updated version.

Recall that in HW 3, you were asked to give an example of functions f and g so that $\lim_{x\to 1} g(x) = 1$, $\lim_{x\to 1} f(x) = 1$, but $\lim_{x\to 1} f \circ g(x) \neq 1$.

1. Let f and g be functions whose domain is $\mathbb{R}$. Suppose that $f(1) = 1$; f is continuous at 1, and $\lim_{x\to 1} g(x) = 1$. Prove that

$$\lim_{x\to 1} f \circ g(x) = 1.$$

2. This problem shows a relationship between one-sided limits and limits at infinity. Let $g(x) = 1/x$. Let $f(x)$ be a function and suppose that $(0, \infty) \subset D(f)$.

a. Suppose that $\lim_{x\to 0^+} f(x) = L$. Prove that $\lim_{x\to\infty} f \circ g(x) = L$ (don't forget to establish *both* requirements for the limit to exist).

b. Suppose that $\lim_{x\to\infty} f \circ g(x) = L$. Prove that $\lim_{x\to 0^+} f(x) = L$.

3. Let f and g be functions, and suppose f and g are continuous (in particular, this means $D(f) = \mathbb{R}$, $D(g) = \mathbb{R}$). Suppose that for every rational number $x \in \mathbb{Q}$, we have $f(x) = g(x)$. Prove that $f(x) = g(x)$ for *every* number $x \in \mathbb{R}$.

4 a. Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$ be a polynomial. Suppose that $a_n > 0$ and $a_0 < 0$. Prove that there exists a point $x \in \mathbb{R}$ with $f(x) = 0$.

b. Give an example of a non-constant polynomial f (i.e. the polynomial must have degree at least one) so that $f(x) \neq 0$ for all $x \in \mathbb{R}$. You do not have to prove that your example is correct.