

Homework 10

1. Section 11.1, #12
2. Suppose R is a symmetric and transitive relation on a set A , and for every $x \in A$ there exists some $a \in A$ such that aRx . Prove that R is reflexive.
3. Let $A = \{1, 2, 3\}$. Give an example of a relation on A that is symmetric and transitive, but not reflexive.
4. Below, we describe four relations on the integers. For each, prove or disprove that it is an equivalence relation. For the equivalence relation(s), describe [26], either by writing out all its terms, or by noticing that it is a familiar set.
 - (a) $Q \subseteq \mathbb{Z} \times \mathbb{Z}$, $Q = \{(a, b) : \gcd(a, b) > 1\}$
 - (b) $R \subseteq \mathbb{Z} \times \mathbb{Z}$, $R = \{(a, b) : |a - b| < 2\}$
 - (c) $S \subseteq \mathbb{Z} \times \mathbb{Z}$, $S = \{(a, b) : a^2 = b^2\}$
 - (d) $T \subseteq \mathbb{Z} \times \mathbb{Z}$, $T = \{(a, b) : a^2 \equiv b^2 \pmod{4}\}$
5. Cigol is a student who dislikes truth tables. Cigol is considering statements that are made out of statements P and Q (possibly repeated), together with the symbols \vee , \wedge , and \sim . Cigol will call two statements “related” if they agree in at least three of the four columns of a truth table. For example: the statements $P \vee Q$ and $(P \vee Q) \wedge \sim (P \wedge Q)$ agree in three cases (P and Q both false; P true and Q false; P false and Q true), so Cigol calls them related.

Show that Cigol’s relation is not an equivalence relation.
6. Section 11.2, #4
7. List all the partitions of $A = \{a, b, c\}$.
8. Let $A = \{0, 1\}^3$: that is, A is the set of all ordered triples with entries from 0 and 1. Then define a relation $R \subseteq A \times A$ such that xRy if and only if x and y have the same number of 0s. Note that R is an equivalence relation.

Give the partition of A created by the equivalence classes of R .
9. Section 11.4, #2 (see Page 192 for examples)
10. Section 11.4, #6