REVIEW PROBLEMS

PROBLEM 1 CALCULATE THE FOLLOWING INTEGRALS USING COMPLEX CONTOUR INTEGRATION:

(i) $I = \int_0^\infty \frac{x^\alpha}{x^2+\sqrt{2}x+1}\,dx \qquad -1<\alpha<1.$ (ii) $I = \int_0^\infty \frac{\ln x}{x^3+1}\,dx$

PROBLEM 2 CONSIDER THE MULTI-VALUED FUNCTION

$$W: f(z) = \left(\frac{z+1}{z-6}\right)^{2/3}$$

(i) DEFINE A BRANCH THAT IS CONTINUOUS AT $z = \pm 7$ WITH $f(7) = 4$ AND EVALUATE $f(-7)$.

(ii) DEFINE A DIFFERENT BRANCH WITH $f(7) = 4$ THAT IS CONTINUOUS AT $z = \pm 7$ AND AT THE ORIGIN.

PROBLEM 3 FIND ALL ROOTS OF

(i) $\cos z = 2i$ (ii) $2i = e^{3z}$

PROBLEM 4 FIND ALL BRANCH POINTS OF

$$f(z) = \log\left(z + \frac{1}{z}\right).$$

DEFINE A BRANCH THAT IS CONTINUOUS ON THE CIRCLE $|z-i| = \pi/2$ WITH $f(2i) = \ln\frac{3}{2} + i\pi/2$

PROBLEM 5 CALCULATE THE FOLLOWING INTEGRAL USING COMPLEX INTEGRATION $\int_0^3 \sqrt{x(3-x)}\,dx.$

PROBLEM 6 FIND THE IMAGE OF THE REGION SHOWN UNDER THE MAP $W = \sin z$